



Theory Reference

Release 5.6

We have shipped this manual unassembled to prevent unnecessary damage to the pages and binder rings during shipping.

To assemble the manual:

1. Remove the shrink-wrap packaging.
2. Insert all pages onto the binder rings. We suggest you keep the heavy, blank pages on the front and back of the manual.
3. Insert the plastic sheet lifter in *front* of the manual to help keep the pages from getting caught under the curved portion of the rings.

ANSYS, Inc.
Southpointe
275 Technology Drive
Canonsburg, PA 15317
Phone: (724) 746-3304
Fax: (724) 514-9494
Internet: ansysinfo@ansys.com

ANSYS

Theory Reference

Release 5.6

Edited by
Peter Kohnke, Ph.D.

ANSYS, Inc.
Southpointe
275 Technology Drive
Canonsburg, PA 15317
Phone: (724) 746-3304
Fax: (724) 514-9494
Internet: ansysinfo@ansys.com

ANSYS, Inc. is a UL registered **ISO 9001: 1994** Company

Copyright© 1992, 1993, 1994, 1995, 1996, 1997, 1998, 1999 SAS IP
All Rights Reserved

Revision History

DN-R300:50-4	Upd0	ANSYS Revision 5.0	December 23, 1992
DN-R300:50-4	Upd0	ANSYS Revision 5.0	November 1, 1993
DN-R300:50-4	Upd0	ANSYS Revision 5.0	November 19, 1993
DN-R300:50-4	Upd0	ANSYS Revision 5.0	February 15, 1994
DN-R300:51-4	Upd0	ANSYS Revision 5.1	September 30, 1994
000450	Upd0	ANSYS Revision 5.2	August 31, 1995
000656	Seventh Edition	ANSYS Release 5.3	June 1996
000855	Eighth Edition	ANSYS Release 5.4	September 1997
001099	Tenth Edition	ANSYS Release 5.5	September 1998
001242	Eleventh Edition	ANSYS Release 5.6	November 1999

Registered Trademarks

ANSYS® is a registered trademark of SAS IP, Inc.

All other product names mentioned in this manual are trademarks or registered trademarks of their respective manufacturers.

Disclaimer Notice

This document has been reviewed and approved in accordance with ANSYS, Inc. Documentation Review and Approval Procedures.

“This ANSYS, Inc. software product (the Program) and program documentation (Documentation) are furnished by ANSYS, Inc. under an ANSYS Software License Agreement that contains provisions concerning non-disclosure, copying, length and nature of use, warranties, disclaimers and remedies, and other provisions. The Program and Documentation may be used or copied only in accordance with the terms of that License Agreement.”

© 1999 SAS IP, Inc. All rights reserved. Unpublished rights reserved under the Copyright Laws of the United States.

U.S. GOVERNMENT RIGHTS

Use, duplication, or disclosure by the U.S. Government is subject to restrictions set forth in the ANSYS, Inc. license agreement and as provided in DFARS 227.7202-1(a) and 227.7202-3(a) (1995), DFARS 252.227-7013(c)(1)(ii) (OCT 1988), FAR 12.212(a) (1995), FAR 52.227-19, or FAR 52.227-14 (ALT III), as applicable. ANSYS, Inc.

THIS SOFTWARE CONTAINS CONFIDENTIAL INFORMATION AND TRADE SECRETS OF SAS IP, INC. USE, DISCLOSURE, OR REPRODUCTION IS PROHIBITED WITHOUT THE PRIOR EXPRESS WRITTEN PERMISSION OF ANSYS, INC.

Acknowledgement

The contributions of the technical staff at ANSYS, Inc. are gratefully acknowledged for writing most of the modifications required to update the Theory volume of the User's Manual from the previous edition. Production help came from many sides, but special mention must be made of Jackie Williamson, who considers this "her" manual for good reason. Further, many people throughout ANSYS, Inc. were involved in checking of the manuscript and this is also appreciated.

Note

Most equations are shown in a different font than the rest of the text. For example, the equation may use ϱ , but the text uses ρ . This discrepancy will be removed at a later edition. The editor apologizes for any inconvenience that this causes.

Future Editions

The ANSYS program is continually being improved and expanded, so future editions of this manual are planned. It is requested that all suggestions for improvements to the manual, or to the theory itself, be forwarded to the editor so this manual and the ANSYS program can be made more useful.

Theory Reference Table of Contents

Title, Disclaimer of Warranty and Liability	i
--	----------

1 Introduction

1.1 Introduction	1-1
------------------------	-----

The Phenomena

2 Structures

2.1 Structural Fundamentals	2-1
2.2 Derivation of Structural Matrices	2-12
2.3 Structural Strain and Stress Evaluations	2-17

3 Structures with Geometric Nonlinearities

3.0 Introduction to Geometric Nonlinearities	3-1
3.1 Large Strain	3-5
3.2 Large Rotation	3-12
3.3 Stress Stiffening	3-23
3.4 Spin Softening	3-31

4 Structures with Material Nonlinearities

4.0 Introduction to Material Nonlinearities	4-1
4.1 Rate-Independent Plasticity	4-5
4.2 Rate-Dependent Plasticity	4-30
4.3 Creep	4-33

4.4	Nonlinear Elasticity	4-37
4.5	Hyperelasticity	4-39
4.6	Viscoelasticity	4-53
4.7	Concrete	4-56
4.8	Swelling	4-65

5 Electromagnetics

5.1	Electromagnetic Field Fundamentals	5-1
5.2	Derivation of Electromagnetic Matrices	5-18
5.3	Electromagnetic Field Evaluations	5-25
5.4	Voltage Forced and Circuit-Coupled Magnetic Field	5-34
5.5	High-Frequency Electromagnetic Simulation	5-37
5.6	Inductance Computation	5-45
5.7	Electromagnetic Particle Tracing	5-48
5.8	Maxwell Stress Tensor	5-50
5.9	Electromechanical Transducer for MEMS	5-55
5.10	Capacitance Computation by CMATRIX Macro	5-56
5.11	Open Boundary Analysis with a Trefftz Domain	5-59

6 Heat Flow

6.1	Heat Flow Fundamentals	6-1
6.2	Derivation of Heat Flow Matrices	6-6
6.3	Heat Flow Evaluations	6-9
6.4	Radiation Matrix Method	6-11
6.5	Radiosity Solution Method	6-16

7 Fluid Flow

7.1	Fluid Flow Fundamentals	7-1
7.2	Derivation of Fluid Flow Matrices	7-19
7.3	Volume of Fluid Method for Free Surface Flows	7-31
7.4	Fluid Solvers	7-39
7.5	Overall Convergence and Stability	7-41

7.6	Fluid Properties	7–45
7.7	Derived Quantities	7–52

8 Acoustics

8.1	Acoustic Fluid Fundamentals	8–1
8.2	Derivation of Acoustics Fluid Matrices	8–4
8.3	Absorption of Acoustical Pressure Wave	8–6
8.4	Acoustics Fluid – Structure Coupling	8–8
8.5	Acoustics Output Quantities	8–10

9 This chapter intentionally omitted

10 This chapter intentionally omitted

11 Coupling

11.0	Coupled Effects	11–1
11.1	Piezoelectrics	11–14
11.2	Thermal–Electric Elements	11–20

The Elements

12 Shape Functions

12.0	Shape Functions	12–1
12.1	2–D Lines	12–3
12.2	3–D Lines	12–4
12.3	Axisymmetric Shells	12–7
12.4	Axisymmetric Harmonic Shells	12–9
12.5	3–D Shells	12–11
12.6	2–D and Axisymmetric Solids	12–22
12.7	Axisymmetric Harmonic Solids	12–29
12.8	3–D Solids	12–33

12.9	Electromagnetic Edge Elements	12–51
------	-------------------------------------	-------

13 Element Tools

13.1	Integration Point Locations	13–1
13.2	Lumped Matrices	13–10
13.3	Reuse of Matrices	13–13
13.4	Temperature–Dependent Material Properties	13–15
13.5	Positive Definite Matrices	13–16
13.6	Nodal and Centroidal Data Evaluation	13–17
13.7	Element Shape Testing	13–18

14 Element Library

14.0	Introduction	14–1
14.1	LINK1 — 2–D Spar (or Truss)	14–3
14.2	PLANE2 — 2–D 6–Node Triangular Structural Solid	14–4
14.3	BEAM3 — 2–D Elastic Beam	14–6
14.4	BEAM4 — 3–D Elastic Beam	14–11
14.5	SOLID5 — 3–D Coupled Field Solid	14–20
14.6	This section intentionally omitted	
14.7	COMBIN7 — Revolute Joint	14–22
14.8	LINK8 — 3–D Spar (or Truss)	14–29
14.9	INFIN9 — 2–D Infinite Boundary	14–34
14.10	LINK10 — Tension Only or Compression–only Spar	14–38
14.11	LINK11 — Linear Actuator	14–42
14.12	CONTAC12 — 2–D Point–to–Point Contact	14–46
14.13	PLANE13 — 2–D Coupled–Field Solid	14–50
14.14	COMBIN14 — Spring–Damper	14–53
14.15	This section intentionally omitted	
14.16	PIPE16 — Elastic Straight Pipe	14–57
14.17	PIPE17 — Elastic Pipe Tee	14–69
14.18	PIPE18 — Elastic Curved Pipe (Elbow)	14–71
14.19	This section intentionally omitted	
14.20	PIPE20 — Plastic Straight Pipe	14–77

14.21	MASS21 — Structural Mass	14–83
14.22	This section intentionally omitted	
14.23	BEAM23 — 2–D Plastic Beam	14–85
14.24	BEAM24 — 3–D Thin–Walled Beam	14–99
14.25	PLANE25 — 4–Node Axisymmetric–Harmonic Structural Solid	14–111
14.26	CONTAC26 — 2–D Point–to–Ground Contact	14–114
14.27	MATRIX27 — Stiffness, Damping, or Mass Matrix	14–117
14.28	SHELL28 — Shear/Twist Panel	14–118
14.29	FLUID29 — 2–D Acoustic Fluid	14–121
14.30	FLUID30 — 3–D Acoustic Fluid	14–122
14.31	LINK31 — Radiation Link	14–123
14.32	LINK32 — 2–D Conduction Bar	14–126
14.33	LINK33 — 3–D Conduction Bar	14–127
14.34	LINK34 — Convection Link	14–129
14.35	PLANE35 — 2–D 6–Node Triangular Thermal Solid	14–131
14.36	SOURC36 — Current Source	14–132
14.37	COMBIN37 — Control	14–133
14.38	FLUID38 — Dynamic Fluid Coupling	14–138
14.39	COMBIN39 — Nonlinear Spring	14–142
14.40	COMBIN40 — Combination	14–148
14.41	SHELL41 — Membrane Shell	14–153
14.42	PLANE42 — 2–D Structural Solid	14–156
14.43	SHELL43 — Plastic Shell	14–158
14.44	BEAM44 — 3–D Elastic Tapered Unsymmetrical Beam	14–165
14.45	SOLID45 — 3–D Structural Solid	14–174
14.46	SOLID46 — 3–D Layered Structural Solid	14–176
14.47	INFIN47 — 3–D Infinite Boundary	14–184
14.48	CONTAC48 — 2–D Point–to–Surface Contact	14–191
14.49	CONTAC49 — 3–D Point–to–Surface Contact	14–201
14.50	MATRIX50 — Superelement (or Substructure)	14–212
14.51	SHELL51 — Axisymmetric Structural Shell	14–214
14.52	CONTAC52 — 3–D Point–to–Point Contact	14–216
14.53	PLANE53 — 2–D 8–Node Magnetic Solid	14–218

14.54	BEAM54 — 2-D Elastic Tapered Unsymmetric Beam	14-221
14.55	PLANE55 — 2-D Thermal Solid	14-223
14.56	HYPHER56 — 2-D 4-Node Mixed U-P Hyperelastic Solid . .	14-226
14.57	SHELL57 — Thermal Shell	14-228
14.58	HYPHER58 — 3-D 8-Node Mixed U-P Hyperelastic Solid . .	14-229
14.59	PIPE59 — Immersed Pipe or Cable	14-234
14.60	PIPE60 — Plastic Curved Pipe (Elbow)	14-249
14.61	SHELL61 — Axisymmetric-Harmonic Structural Shell	14-257
14.62	SOLID62 — 3-D Coupled Magnetic-Structural Solid	14-263
14.63	SHELL63 — Elastic Shell	14-265
14.64	SOLID64 — 3-D Anisotropic Structural Solid	14-271
14.65	SOLID65 — 3-D Reinforced Concrete Solid	14-273
14.66	This section intentionally omitted	
14.67	PLANE67 — 2-D Coupled Thermal-Electric Solid	14-284
14.68	LINK68 — Coupled Thermal-Electric Line	14-285
14.69	SOLID69 — 3-D Coupled Thermal-Electric Solid	14-286
14.70	SOLID70 — 3-D Thermal Solid	14-287
14.71	MASS71 — Thermal Mass	14-290
14.72	SOLID72 — 4-Node Tetrahedral Structural Solid with Rotations	14-292
14.73	SOLID73 — 3-D 8-Node Structural Solid with Rotations . . .	14-294
14.74	HYPHER74 — 2-D 8-Node Mixed U-P Hyperelastic Solid . .	14-296
14.75	PLANE75 — Axisymmetric-Harmonic Thermal Solid	14-298
14.76	This section intentionally omitted	
14.77	PLANE77 — 2-D 8-Node Thermal Solid	14-299
14.78	PLANE78 — Axisymmetric-Harmonic 8-Node Thermal Solid	14-301
14.79	FLUID79 — 2-D Contained Fluid	14-303
14.80	FLUID80 — 3-D Contained Fluid	14-305
14.81	FLUID81 — Axisymmetric-Harmonic Contained Fluid	14-311
14.82	PLANE82 — 2-D 8-Node Structural Solid	14-313
14.83	PLANE83 — 8-Node Axisymmetric-Harmonic Structural Solid	14-315
14.84	HYPHER84 — 2-D 8-Node Hyperelastic Solid	14-317
14.85	This section intentionally omitted	

14.86	HYPER86 — 3-D 8-Node Hyperelastic Solid	14-319
14.87	SOLID87 — 3-D 10-Node Tetrahedral Thermal Solid	14-326
14.88	VISCO88 — 2-D 8-Node Viscoelastic Solid	14-327
14.89	VISCO89 — 20-Node Viscoelastic Solid	14-329
14.90	SOLID90 — 20-Node Thermal Solid	14-331
14.91	SHELL91 — Nonlinear Layered Structural Shell	14-333
14.92	SOLID92 — 3-D 10-Node Tetrahedral Structural Solid	14-340
14.93	SHELL93 — 8-Node Structural Shell	14-342
14.94	This section intentionally omitted	
14.95	SOLID95 — 20-Node Structural Solid	14-345
14.96	SOLID96 — 3-D Magnetic Scalar Solid	14-347
14.97	SOLID97 — 3-D Magnetic Solid	14-348
14.98	SOLID98 — Tetrahedral Coupled-Field Solid	14-350
14.99	SHELL99 — Linear Layered Structural Shell	14-352
14.100	This section intentionally omitted	
14.101	This section intentionally omitted	
14.102	This section intentionally omitted	
14.103	This section intentionally omitted	
14.104	This section intentionally omitted	
14.105	This section intentionally omitted	
14.106	VISCO106 — 2-D Viscoplastic Solid	14-365
14.107	VISCO107 — 3-D Viscoplastic Solid	14-367
14.108	VISCO108 — 2-D 8-Node Viscoplastic Solid	14-371
14.109	This section intentionally omitted	
14.110	INFIN110 — 2-D Infinite Solid	14-373
14.111	INFIN111 — 3-D Infinite Solid	14-378
14.112	This section intentionally omitted	
14.113	This section intentionally omitted	
14.114	This section intentionally omitted	
14.115	INTER115 — 3-D Magnetic Interface	14-379
14.116	FLUID116 — Coupled Thermal-Fluid Pipe	14-386
14.117	SOLID117 — 3-D Magnetic Edge	14-394
14.118	This section intentionally omitted	
14.119	HF119 — 3-D High-Frequency Tetrahedral Solid	14-397

14.120	HF120 — High-Frequency Brick Solid	14-400
14.121	PLANE121 — 2-D 8-Node Electrostatic Solid	14-405
14.122	SOLID122 — 20-Node Electrostatic Solid	14-406
14.123	SOLID123 — 3-D 10-Node Tetrahedral Electrostatic Solid .	14-407
14.124	CIRCU124 — General Electric Circuit Element	14-408
14.125	This section intentionally omitted	
14.126	TRANS126 — Electromechanical Transducer for MEMS ...	14-411
14.127	SOLID127 — Tet Electrostatic p-Element	14-415
14.128	SOLID128 — Brick Electrostatic p-Element	14-416
14.129	FLUID129 — 2-D Infinite Acoustic	14-418
14.130	FLUID130 — 3-D Infinite Acoustic	14-419
14.131	This section intentionally omitted	
14.132	This section intentionally omitted	
14.133	This section intentionally omitted	
14.134	This section intentionally omitted	
14.135	This section intentionally omitted	
14.136	This section intentionally omitted	
14.137	This section intentionally omitted	
14.138	This section intentionally omitted	
14.139	This section intentionally omitted	
14.140	This section intentionally omitted	
14.141	FLUID141 — 2-D Fluid	14-425
14.142	FLUID142 — 3-D Fluid	14-428
14.143	SHELL143 — Plastic Shell	14-434
14.144	This section intentionally omitted	
14.145	PLANE145 — 2-D Quadrilateral Structural Solid p-Element	14-438
14.146	PLANE146 — 2-D Triangular Structural Solid p-Element ..	14-440
14.147	SOLID147 — 3-D Brick Structural Solid p-Element	14-442
14.148	SOLID148 — 3-D Tetrahedral Structural Solid p-Element .	14-444
14.149	This section intentionally omitted	
14.150	SHELL150 — 8-Node Structural Shell p-Element	14-446
14.151	SURF151 — 2-D Thermal Surface Effect	14-449
14.152	SURF152 — 3-D Thermal Surface Effect	14-450
14.153	SURF153 — 2-D Structural Surface Effect	14-455

14.154	SURF154 — 3-D Structural Surface Effect	14-456
14.155	This section intentionally omitted	
14.156	This section intentionally omitted	
14.157	SHELL157 — Coupled Thermal-Electric Shell	14-460
14.158	HYPHER158 — 3-D 10-Node Tetrahedral Mixed U-P Hyperelastic Solid	14-461
14.159	This section intentionally omitted	
14.160	LINK160 — Explicit 3-D Spar	14-463
14.161	BEAM161 — Explicit 3-D Beam	14-465
14.162	This section intentionally omitted	
14.163	SHELL163 — Explicit Thin Structural Shell	14-466
14.164	SOLID164 — Explicit 3-D Structural Solid	14-467
14.165	COMBI165 — Explicit Spring-Damper	14-468
14.166	MASS166 — Explicit 3-D Structural Mass	14-469
14.167	LINK167 — Explicit Tension-Only Spar	14-470
14.168	This section intentionally omitted	
14.169	TARGE169 — 2-D Target Segment	14-471
14.170	TARGE170 — 3-D Target Segment	14-472
14.171	CONTA171 — 2-D Surface-to-Surface Contact	14-474
14.172	CONTA172 — 2-D 3-Node Surface-to-Surface Contact ...	14-475
14.173	CONTA173 — 3-D Surface-to-Surface Contact	14-476
14.174	CONTA174 — 3-D 8-Node Surface-to-Surface Contact ...	14-477
14.175	This section intentionally omitted	
14.176	This section intentionally omitted	
14.177	This section intentionally omitted	
14.178	This section intentionally omitted	
14.179	PRETS179 — Pre-tension	14-483
14.180	LINK180 — 3-D Finite Strain Spar (or Truss)	14-484
14.181	SHELL181 — Large Strain Shell	14-487
14.182	PLANE182 — 2-D Structural Solid	14-490
14.183	PLANE183 — 2-D 8-Node Structural Solid	14-492
14.184	This section intentionally omitted	
14.185	SOLID185 — 3-D Structural Solid	14-494
14.186	SOLID186 — 20-Node Structural Solid	14-496

14.187	SOLID187 — 3-D 10-Node Tetrahedral Structural Solid . . .	14-498
14.188	BEAM188 — 3-D Finite Strain Linear Beam	14-500
14.189	BEAM189 — 3-D Finite Strain Quadratic Beam	14-502

The Solvers

15 Analysis Tools

15.1	Acceleration Effect	15-1
15.2	Inertia Relief	15-3
15.3	Damping Matrices	15-8
15.4	Element Reordering	15-10
15.5	Automatic Master DOF Selection	15-12
15.6	Automatic Time Stepping	15-13
15.7	Wavefront Solver	15-18
15.8	Constraint Equations	15-23
15.9	Newton-Raphson Procedure	15-28
15.12	Eigenvalue and Eigenvector Extraction	15-41
15.13	Mode Superposition Method	15-55
15.12	Solving for Unknowns and Reactions	15-60
15.13	Conjugate Gradient Solvers	15-65
15.14	Modal Analysis of Cyclic Symmetric Structures	15-68
15.15	Mass Moments of Inertia	15-71
15.16	Energies	15-75
15.15	Mass Moments of Inertia	15-69
15.16	Energies	15-73

16 This chapter intentionally omitted

17 Analysis Procedures

17.0	Analysis Procedures	17-1
17.1	Static Analysis (ANTYPE,STATIC)	17-2
17.2	Transient Analysis (ANTYPE,TRANS)	17-5

17.3	Mode–Frequency Analysis (ANTYPE,MODAL)	17–17
17.4	Harmonic Response Analyses (ANTYPE, HARMIC)	17–19
17.5	Buckling Analysis (ANTYPE, BUCKLE)	17–28
17.6	Substructuring Analysis (ANTYPE, SUBSTR)	17–30
17.7	Spectrum Analysis (ANTYPE,SPECTR)	17–34

The Data Handlers

18 Pre and Postprocessing Tools

18.1	Integration and Differentiation Procedures	18–1
18.2	Fourier Coefficient Evaluation	18–4
18.3	Statistical Procedures	18–7

19 Postprocessing

19.1	POST1 — Derived Nodal Data Processing	19–1
19.2	POST1 — Vector and Surface Operations	19–3
19.3	POST1 — Path Operations	19–5
19.4	POST1 — Stress Linearization	19–10
19.5	POST1 — Fatigue Module	19–21
19.6	POST1 — Electromagnetic Macros	19–24
19.7	POST1 — Error Approximation Technique	19–39
19.8	POST1 — Crack Analysis (KCALC Command)	19–44
19.9	POST1 — Harmonic Solid and Shell Element Postprocessing	19–48
19.10	POST26 — Data Operations	19–51
19.11	POST26 — Response Spectrum Generator (RESP)	19–53
19.12	POST1 and POST26 — Interpretation of Equivalent Strains	19–56
19.13	POST26 — Response Power Spectral Density (RPSD)	19–58
19.14	POST26 — Computation of Covariance (CVAR)	19–59

20 Design Optimization

20.0	Introduction to Optimization	20–1
------	------------------------------------	------

20.1	Introduction to Design Optimization	20-2
20.2	Single-Loop Analysis Tool	20-5
20.3	Random Tool	20-6
20.4	Sweep Tool	20-7
20.5	Factorial Tool	20-8
20.6	Gradient Tool	20-9
20.7	Subproblem Approximation Method	20-10
20.8	First Order Optimization Method	20-16
20.9	Topological Optimization	20-20

References

Index