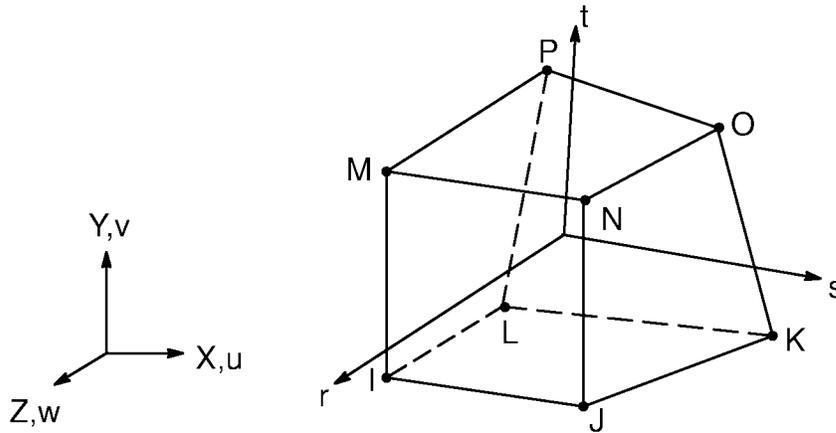


14.107 VISCO107 — 3-D Viscoplastic Solid



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equations (12.8.18-1), (12.8.18-2) and (12.8.18-3)	2 x 2 x 2
Mass Matrix	Same as stiffness matrix	2 x 2 x 2
Thermal Load Vector	Same as stiffness matrix	2 x 2 x 2
Pressure Load Vector	Same as stiffness matrix, specialized to the face	2 x 2

Load Type	Distribution
Element Temperature	Trilinear thru element
Nodal Temperature	Trilinear thru element
Pressure	Bilinear across each face

References: Oden(123), Weber et al.(127)

14.107.1 Basic Assumptions

This section discusses the basic theory of the large strain viscoplastic elements. The elements developed use the updated Lagrangian concept along with logarithmic (Hencky) strain and Cauchy (True) stress measures. The material is limited to be isotropic in nature and elastic strains are assumed to be small relative to plastic strains. Further the plastic flow is assumed to be isochoric (i.e. volume preserving) and both the rate-independent and rate-dependent elastic-plastic constitutive relationship is considered.

The strain energy calculation is based on integration of loading rates and a large time increment may produce inaccurate energy values, even though stress-strain solutions are quite accurate.

14.107.2 Element Tangent Matrices and Newton-Raphson Restoring Force

The formulation considered is highly nonlinear in nature both from the point of view of kinematic or geometric consideration as well as constitutive behavior. The full Newton-Raphson solution option is utilized when the Newton-Raphson restoring force is given by (see equation (15.9-2):

$$\{F^{nr}\} = \int_{vol} [B]^T \{\sigma\} d(vol) \quad (14.107-1)$$

where: $[B]$ = strain-displacement matrix
 $\{\sigma\}$ = Cauchy stress

Equation (14.107-1) is modified by assuming a decomposition of the Cauchy stress into the deviatoric part plus the pressure part:

$$\{\sigma\} = \{\sigma'\} - \{q\}P \quad (14.107-2)$$

where: $\{\sigma'\}$ = Cauchy stress deviator
 $[q]$ = $\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$
 P = hydrostatic stress = $-(\sigma_x + \sigma_y + \sigma_z) / 3$

The negative value of P is output as HPRES.

The pressure is separately interpolated to conveniently allow for enforcement of the incompressibility constraint associated with large plastic strains (Oden and Kikuchi(123)). The restoring force can now be rewritten as:

$$\{F^{nr}\} = \int_{vol} [B]^T \{\sigma'\} d(vol) - \int [B]^T \{q\} P d(vol) \quad (14.107-3)$$

The incompressibility constraint during plastic flow is enforced through the augmentation of the momentum equations with the additional equation:

$$\int_{vol} [N^P]^T (\Delta J - \Delta \hat{J} (\Delta P)) d(vol) = 0 \quad (14.107-4)$$

where:

- $[N^P]$ = shape function associated with the independently interpolated pressure DOF
- ΔJ = determinant of the relative deformation gradient (the relative volume change)
- $\Delta \hat{J}$ = constitutively prescribed function expressing the pressure–volume change relationship and expressed as:

$$\Delta \hat{J} = \exp \frac{-\Delta P}{K} \quad (14.107-5)$$

where:

- K = elastic bulk modulus for the material (= $E / (3(1-2\nu))$)
- E = Young's modulus (input as EX on **MP** command)
- ν = Poisson's ratio (input as NUXY on **MP** command)

The total Cauchy stress is calculated by finding the deviatoric part from the constitutive equations using the strains calculated from nodal displacements and subtracting the separately interpolated pressure, i.e.:

$$\{\bar{\sigma}\} = \{\sigma'\} - \{q\} P_o \quad (14.107-6)$$

where: P_o = interpolated from the pressure field

The stiffness matrix for the static analysis is constructed by evaluating the exact Jacobian of the discretized system. This yields an equation of the form:

$$\begin{bmatrix} K^{uu} & K^{up} \\ K^{pu} & K^{pp} \end{bmatrix} \begin{Bmatrix} \Delta u \\ \Delta P \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix} - \begin{Bmatrix} F^u \\ F^p \end{Bmatrix} \quad (14.107-7)$$

where:

- $\{F\}$ = external nodal forces
- $\{\Delta u\}, \{\Delta P\}$ = increments of displacement and pressure, respectively

$$\{F^u\} = \int_{vol} [B]^T \{\bar{\sigma}\} d(vol)$$

$$\begin{aligned} \{F^p\} &= \int_{\text{vol}} [N^p]^T (\Delta J - \Delta \hat{J}(\Delta P)) d(\text{vol}) \\ [K^{uu}] &= \frac{\partial}{\partial u} \left[\int_{\text{vol}} [B]^T \{\bar{\sigma}\} d(\text{vol}) \right] \\ [K^{up}] &= [K^{pu}]^T = \frac{\partial}{\partial p} \left[\int_{\text{vol}} [B]^T \{\bar{\sigma}\} d(\text{vol}) \right] \\ [K^{pp}] &= \frac{\partial}{\partial p} \left[\int_{\text{vol}} [N^p] (\Delta J - \Delta \hat{J}(\Delta P)) d(\text{vol}) \right] \end{aligned}$$

The tangent matrix developed in equation (14.107–7) has two parts, namely the constitutive part and geometric part. From the requirement of full tangent matrices, both the constitutive and geometric parts are essential, but the numerical efficiency and stability considerations can prove to be different. Thus, it is left on the user to control the inclusion of stress (geometric) stiffness by the **SSTIF,ON** or **SSTIF,OFF** command. Symmetry of the stiffness matrix is achieved by assuming small strain increments for the constitutive part and negligible volume change during the step for the geometric part. The assumptions generally result in good convergence characteristic for these elements even when these assumptions of small strain increments and negligible volume change are violated. Additional detail of the stiffness matrix can be found in Weber et al.(127).

For the constitutive part of the rate-dependent plasticity (Anand's model), see Section 4.2. Section 13.1 describes the integration point locations.

14.107.3 Plastic Energy Output

$$\begin{aligned} E_c^{pl} &= \sum_{i=1}^{NINT} \sum_{j=1}^{NCS} \{\sigma\}^T \{\Delta \epsilon^{pl}\} \text{vol}_i \\ &= \text{plastic energy per unit volume (accessed with NL,PLWK} \\ &\quad \text{on the **ETABLE** command (VISCO106, VISCO107 and} \\ &\quad \text{VISCO108 only))} \end{aligned} \tag{14.107–8}$$

- NINT = number of integration points
- NCS = total number of converged substeps
- $\{\Delta \epsilon^{pl}\}$ = plastic strain increment