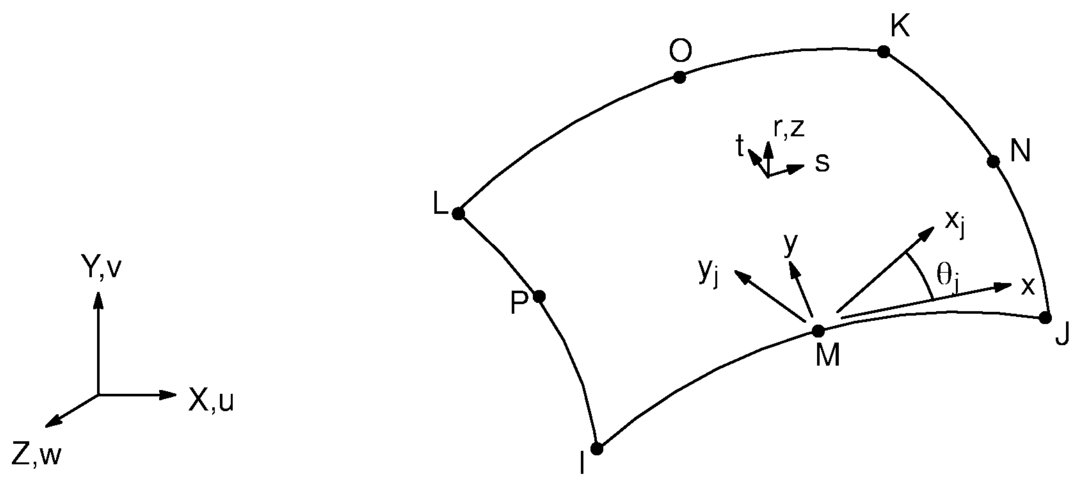
14.99 SHELL99 — Linear Layered Structural Shell



Matrix or Vector	Geometry	Shape Functions	Integration Points
Stiffness Matrix	Quad	Equations (12.5.14–1), (12.5.14–2) and (12.5.14–3)	Thru the thickness: 2 In–plane: 2 x 2
	Triangle	Equations (12.5.5–1), (12.5.5–2) and (12.5.5–3)	Thru the thickness: 2 In–plane: 3
Mass Matrix	Quad	Equations (12.5.10–1), (12.5.10–2) and (12.5.10–3)	Same as stiffness matrix
	Triangle	Equations (12.5.2–1), (12.5.2–2) and (12.5.2–3)	Same as stiffness matrix
Stress Stiffness Matrix	Same as mass matrix		Same as stiffness matrix
Thermal Load Vector	Same as stiffness matrix		Same as stiffness matrix

Matrix or Vector	Geometry	Shape Functions	Integration Points
Transverse Pressure Load Vector	Quad	Equation (12.5.10–3)	2 x 2
	Triangle	Equation (12.5.2–3)	3
Edge Pressure Load Vector	Same as in–plane mass matrix, specialized to the edge		2

Load Type	Distribution
Element Temperature	Linear thru thickness, bilinear in plane of element
Nodal Temperature	Constant thru thickness, bilinear in plane of element
Pressure	Bilinear in plane of element, linear along each edge

References: Ahmad(1), Cook(5), Yunus et al(139)

14.99.1 Other Applicable Sections

Chapter 2 describes the derivation of structural element matrices and load vectors as well as stress evaluations. Section 13.1 describes integration point locations. The mass matrix is diagonalized as described in Section 13.2.

14.99.2 Assumptions and Restrictions

Normals to the centerplane are assumed to remain straight after deformation, but not necessarily normal to the centerplane.

Each pair of integration points (in the r direction) is assumed to have the same material orientation.

There is no significant stiffness associated with rotation about the element r axis. A nominal value of stiffness is present using the approach of Zienkiewicz(39), however, to prevent free rotation at the node.

This element does not generate a consistent mass matrix; only the lumped mass matrix is available.

14.99.3 Direct Matrix Input

SHELL99 has two options for the direct input of the matrices that account for the stiffness and mass effects as well as one thermal load distribution. This permits the user to incorporate the results of their own composite material programs, as well as lifting any restriction as to the number of layers.

If KEYOPT(2) = 3, the matrices [E_0], [E_1], [E_2], [E_3], and [E_4] are input directly as input quantities A, B, D, E, and F, respectively on the **R** and **RMORE** commands. For the thermal load, the vectors {S_0}, {S_1}, and {S_2} are input directly as input quantities MT, BT, and QT on the **R** and **RMORE** commands. If KEYOPT(2) = 2, [E_3], [E_4], and {QT} are not used. Further, for both cases, the average density is input directly as input quantity AVDENS on the **RMORE** command.

Considering the KEYOPT(2) = 2 case for a flat shell, the thru thickness accumulated effects can be derived following the theoretical formulation given in reference (139) as:

$$[E_0] = \sum_{j=1}^{N_\ell} \int_{r_j^{bt}}^{r_j^{tp}} [T_m]_j^T [D]_j [T_m]_j \, dr \tag{14.99–1}$$

$$[E_1] = \sum_{j=1}^{N_\ell} \int_{r_j^{bt}}^{r_j^{tp}} r[T_m]_j^T [D]_j [T_m]_j \, dr \tag{14.99–2}$$

$$[E_2] = \sum_{j=1}^{N_\ell} \int_{r_j^{bt}}^{r_j^{tp}} r^2[T_m]_j^T [D]_j [T_m]_j \, dr \tag{14.99–3}$$

$$\{S_0\} = \sum_{j=1}^{N_\ell} \int_{r_j^{bt}}^{r_j^{tp}} [T_m]_j^T [D]_j \{\epsilon^{th}\}_j \, dr \tag{14.99–4}$$

$$\{S_1\} = \sum_{j=1}^{N_\ell} \int_{r_j^{bt}}^{r_j^{tp}} r[T_m]_j^T [D]_j \left\{\epsilon^{th}\right\}_j \, dr \qquad (14.99\text{–}5)$$

where:

N_ℓ = number of layers

$[D]_j$ = stress–strain relationships at point of interest within layer j

$[T_m]$ = layer to element transformation matrix

The previous five definitions can be used to define the forces and moments on a unit square out of the flat shell:

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [E_0] & [E_1] \\ [E_1] & [E_2] \end{bmatrix} \begin{Bmatrix} \{\epsilon\} \\ \{\kappa\} \end{Bmatrix} + \begin{Bmatrix} \{S_0\} \\ \{S_1\} \end{Bmatrix} \qquad (14.99\text{–}6)$$

where:

$\{N\}$ = forces per unit length

$\{M\}$ = moments per unit length

$\{\varepsilon\}$ = strains

$\{\varkappa\}$ = curvatures

Each of the above matrices and load vectors are of sizes 6 x 6 and 6 x 1, as opposed to the 3 x 3 and 3 x 1 sizes commonly used in thin shell analysis. Thus, if only 3 x 3 matrix information is available, it is recommended to input the 6 x 6 matrices in the following form (using $[E_0]$ as an example):

$$[E_o] = \begin{bmatrix} G_{11} & G_{12} & 0 & G_{13} & 0 & 0 \\ G_{12} & G_{22} & 0 & G_{23} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ G_{13} & G_{23} & 0 & G_{33} & 0 & 0 \\ 0 & 0 & 0 & 0 & H & 0 \\ 0 & 0 & 0 & 0 & 0 & H \end{bmatrix} \qquad (14.99\text{–}7)$$

where:

G = 3 x 3 matrix of terms available from outside of the ANSYS program

H = relatively large number to suppress shear deflections (perhaps 10^3 x G_{33})

As discussed earlier, the values in $[E_0]$ (as well as other matrices) used by the ANSYS program for either the layer or matrix input may be printed with KEYOPT(10) = 1 in order to verify the input.

For matrix input, the required stress vector $\{N_c\}$ needed for stress stiffening is computed as:

$$\{N_c\} = \left([E_0]\,[B_0] + [E_1]\,[B_1]\right)\{\delta\} + \{S_0\} \qquad (14.99–8)$$

where: $\{\delta\}$ = $\{u_e\}$ from the previous iteration

14.99.4 Stress Calculations

Strains and stresses are computed at the top and bottom of each layer (KEYOPT(9) = 0) or at the midthickness (KEYOPT(9) = 1). The strains within layer j are:

$$\{\epsilon\}_j = [T_m]_j[B]\,\{u_e\} \qquad (14.99–9)$$

where: $\{u_e\}$ = element displacement vector

The stresses within layer j are:

$$\{\sigma\}_j = [D]_j\left(\{\epsilon\}_j - \{\epsilon^{th}\}_j\right) \qquad (14.99–10)$$

where: $\{\varepsilon^{th}\}_j$ = thermal strain in layer j

14.99.5 Force and Moment Summations

First, all stresses are converted from the layer orientation to the element orientation:

$$\{\sigma_e\}_j = [T_m]_j^T\{\sigma\}_j \qquad (14.99–11)$$

where: $\{\sigma_e\}_j$ = stresses in element orientation

To simplify the below descriptions, the subscript e is dropped. The in–plane forces are computed as:

$$T_x = \sum_{j=1}^{N_\ell} t_j\left[\frac{\sigma^t_{x,j} + \sigma^b_{x,j}}{2}\right] \qquad (14.99–12)$$

$$T_y = \sum_{j=1}^{N_\ell} t_j\left[\frac{\sigma^t_{y,j} + \sigma^b_{y,j}}{2}\right] \qquad (14.99–13)$$

$$T_{xy} = \sum_{j=1}^{N_\ell} t_j \left[\frac{\sigma^t_{xy,j} + \sigma^b_{xy,j}}{2} \right] \tag{14.99–14}$$

where, typically, T_x = output quantity TX

$\sigma^t_{x,j}$ = stress at top of layer j in element x direction

$\sigma^b_{x,j}$ = stress at bottom of layer j in element x direction

t_j = thickness of layer j

The out–of–plane moments are computed as:

$$M_x = \frac{1}{6} \sum_{j=1}^{N_\ell} t_j \left(\sigma^b_{x,j} \left(2z^b_j + z^t_j \right) + \sigma^t_{x,j} \left(2z^t_j + z^b_j \right) \right) \tag{14.99–15}$$

$$M_y = \frac{1}{6} \sum_{j=1}^{N_\ell} t_j \left(\sigma^b_{y,j} \left(2z^b_j + z^t_j \right) + \sigma^t_{y,j} \left(2z^t_j + z^b_j \right) \right) \tag{14.99–16}$$

$$M_{xy} = \frac{1}{6} \sum_{j=1}^{N_\ell} t_j \left(\sigma^b_{xy,j} \left(2z^b_j + z^t_j \right) + \sigma^t_{xy,j} \left(2z^t_j + z^b_j \right) \right) \tag{14.99–17}$$

where, typically, M_x = output quantity MX

z^b_j = z coordinate of bottom layer j

z^t_j = z coordinate of top of layer j

z = coordinate normal to shell, with z=0 being at shell midsurface

The transverse shear forces are computed as:

$$N_x = \sum_{j=1}^{N_\ell} t_j \quad \sigma_{xz,j} \tag{14.99–18}$$

$$N_y = \sum_{j=1}^{N_\ell} t_j \quad \sigma_{yz,j} \tag{14.99–19}$$

where, typically, N_x = output quantity NX

$\sigma_{xz,j}$ = average transverse shear stress in layer j in element x–z plane

For this computation of transverse shear forces, the shear stresses have not been adjusted as shown in the next subsection.

14.99.6 Shear Strain Adjustment

The shape functions assume that the transverse shear strains are constant thru the thickness. However, these strains must be zero at the free surface. Therefore, they are adjusted by:

$$\epsilon'_{xz,j} = \frac{3}{2}\left(1 - r^2\right)\epsilon_{xz,j} \qquad (14.99\text{–}20)$$

$$\epsilon'_{yz,j} = \frac{3}{2}\left(1 - r^2\right)\epsilon_{yz,j} \qquad (14.99\text{–}21)$$

where typically, $\epsilon'_{xz,j}$ = adjusted value of transverse shear strain

$\epsilon_{xz,j}$ = transverse shear strain as computed from strain–displacement relationships

r = normal coordinate, varying from –1.0 (bottom) to +1.0 (top)

Even with this adjustment, these strains will not be exact due to the variable nature of the material properties thru the thickness. However, for thin shell environments, these strains and their resulting stresses are small in comparison to the x, y, and xy components. The interlaminar shear stresses are equivalent to the transverse shear stresses at the layer boundaries and are computed using equilibrium considerations, and hence are more accurate for most applications.

14.99.7 Interlaminar Shear Stress Calculations

In the absence of body forces, the in–plane equilibrium equations of infinitesimally small volume are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} + \frac{\partial \sigma_{xz}}{\partial z} = 0 \qquad (14.99\text{–}22)$$

$$\frac{\partial \sigma_{yx}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \sigma_{yz}}{\partial z} = 0 \qquad (14.99\text{–}23)$$

Rewriting these in incremental form,

$$\Delta\sigma_{xz} = -\Delta z\left(\frac{\Delta\sigma_x}{\Delta x} + \frac{\Delta\sigma_{xy}}{\Delta y}\right) \qquad (14.99\text{–}24)$$

$$\Delta\sigma_{yz} = -\Delta z \left(\frac{\Delta\sigma_{yx}}{\Delta x} + \frac{\Delta\sigma_{y}}{\Delta y} \right) \qquad (14.99\text{–}25)$$

Setting these equations in terms of layer j,

$$\Delta\sigma_{xz,j} = -t_j \left(\frac{\Delta\sigma_{x,j}}{\Delta x} + \frac{\Delta\sigma_{xy,j}}{\Delta y} \right) \qquad (14.99\text{–}26)$$

$$\Delta\sigma_{yz,j} = -t_j \left(\frac{\Delta\sigma_{yx,j}}{\Delta x} + \frac{\Delta\sigma_{y,j}}{\Delta y} \right) \qquad (14.99\text{–}27)$$

where:

$\Delta\sigma_{x,j} = \left(\sigma^2_{x,j} + \sigma^3_{x,j} - \sigma^1_{x,j} - \sigma^4_{x,j}\right)/2.0$

$\Delta\sigma_{xy,j} = \left(\sigma^3_{xy,j} + \sigma^4_{xy,j} - \sigma^1_{xy,j} - \sigma^2_{xy,j}\right)/2.0$

$\Delta\sigma_{yx,j} = \left(\sigma^2_{xy,j} + \sigma^3_{xy,j} - \sigma^1_{xy,j} - \sigma^4_{xy,j}\right)/2.0$

$\Delta\sigma_{y,j} = \left(\sigma^3_{y,j} + \sigma^4_{y,j} - \sigma^1_{y,j} - \sigma^2_{y,j}\right)/2.0$

$\sigma^3_{x,j}$ = stress in element x direction in layer j at integration point 3

Δx and Δy are shown in Figure 14.99–1.

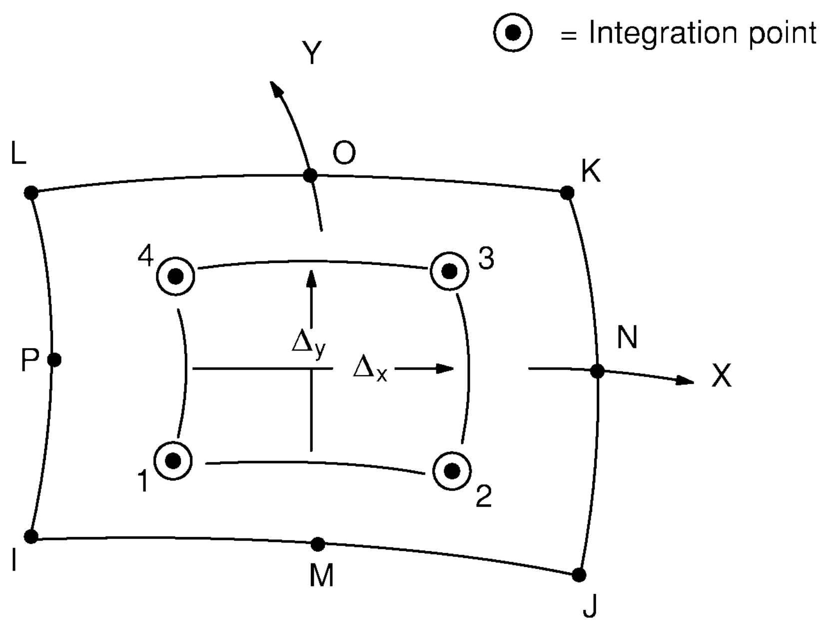


Figure 14.99–1 Integration Point Locations

The interlaminar shear stress components between layer k and layer k+1 may now be written as:

$$\tau_x^k = \sum_{j=1}^{k} \Delta\sigma_{xz,j} - A_x \sum_{j=1}^{k} t_j \qquad (14.99\text{–}28)$$

$$\tau_y^k = \sum_{j=1}^{k} \Delta\sigma_{yz,j} - A_y \sum_{j=1}^{k} t_j \qquad (14.99\text{–}29)$$

where, typically, τ_x^k = output quantity ILSXZ

$A_x = \dfrac{\sum_{j=1}^{N} \Delta\sigma_{xz,j}}{t}$ (= correction term)

t = total thickness

Finally,

$$\tau^k = \sqrt{\left(\tau_x^k\right)^2 + \left(\tau_y^k\right)^2} \qquad (14.99\text{–}30)$$

where: τ^k = output quantity ILSUM

The maximum of all values τ^k is τ_{max}^k (output quantity ILMAX). If τ_{max}^k is less than a small number β, the interlaminar shear stress printout is suppressed and the postdata values are set to zero. β is determined by:

$$\beta = 10^{-8} \sum \left(|\sigma_x| + |\sigma_y| + |\tau_{xy}|\right) \qquad (14.99\text{–}31)$$

where the summation is over all integration points in the top and bottom layers (or in layers LP1 and LP2, if requested).

Finally, a check is made on the validity of the interlaminar shear stresses. R is defined as:

$$R = \frac{t\sqrt{A_x^2 + A_y^2}}{\tau_{max}^k} \qquad (14.99\text{–}32)$$

where: R = output quantity Max. adjustment / Max. stress

R is output if it is greater than 0.1.

14.99.8 Failure Criteria

Possible failure of a material can be evaluated by up to six different criteria, of which three are predefined. They are evaluated at the top and bottom (or middle) of each layer at each of the in–plane integration points. The failure criteria are:

Maximum strain failure criteria

$$\xi_1 = \text{maximum of} \begin{cases} \dfrac{\epsilon_{xt}}{\epsilon^f_{xt}} \text{ or } \dfrac{\epsilon_{xc}}{\epsilon^f_{xc}} & \text{whichever is applicable} \\ \dfrac{\epsilon_{yt}}{\epsilon^f_{yt}} \text{ or } \dfrac{\epsilon_{yc}}{\epsilon^f_{yc}} & \text{whichever is applicable} \\ \dfrac{\epsilon_{zt}}{\epsilon^f_{zt}} \text{ or } \dfrac{\epsilon_{zc}}{\epsilon^f_{zc}} & \text{whichever is applicable} \\ \dfrac{|\epsilon_{xy}|}{\epsilon^f_{xy}} \\ \dfrac{|\epsilon_{yz}|}{\epsilon^f_{yz}} \\ \dfrac{|\epsilon_{xz}|}{\epsilon^f_{xz}} \end{cases} \qquad (14.99\text{–}33)$$

where:

ξ_1 = value of maximum strain failure criterion (output quantity FC1) (accessed with input on **TBDATA** command with **TB**,FAIL)

ε_{xt} = $\begin{cases} 0 \\ \epsilon_x \end{cases}$ whichever is greater

ε_x = strain in layer x–direction

ε_{xc} = $\begin{cases} \epsilon_x \\ 0 \end{cases}$ whichever is lesser

ϵ^f_{xt} = failure strain in layer x–direction in tension (input on **TBDATA** command with **TB**,FAIL)

Maximum stress failure criteria

$$\xi_2 = \text{maximum of} \begin{cases} \dfrac{\sigma_{xt}}{\sigma^f_{xt}} \text{ or } \dfrac{\sigma_{xc}}{\sigma^f_{xc}} & \text{whichever is applicable} \\ \dfrac{\sigma_{yt}}{\sigma^f_{yt}} \text{ or } \dfrac{\sigma_{yc}}{\sigma^f_{yc}} & \text{whichever is applicable} \\ \dfrac{\sigma_{zt}}{\sigma^f_{zt}} \text{ or } \dfrac{\sigma_{zc}}{\sigma^f_{zc}} & \text{whichever is applicable} \\ \dfrac{|\sigma_{xy}|}{\sigma^f_{xy}} \\ \dfrac{|\sigma_{yz}|}{\sigma^f_{yz}} \\ \dfrac{|\sigma_{xz}|}{\sigma^f_{xz}} \end{cases} \quad (14.99\text{–}34)$$

where:

ξ_2 = value of maximum stress failure criterion (output quantity FC2)

$\sigma_{xt} = \begin{cases} 0 \\ \sigma_x \end{cases}$ whichever is greater

σ_x = stress in layer x–direction

$\sigma_{xc} = \begin{cases} \sigma_x \\ 0 \end{cases}$ whichever is lesser

σ^f_{xt} = failure stress in layer x–direction in tension (input on **TBDATA** command with **TB**,FAIL)

Tsai–Wu failure criteria

If TBDATA,3,1 is used after TBTEMP,,CRIT, the criterion used is the “strength index”:

$$\xi_3 = A + B \quad (14.99\text{–}35)$$

and if TBDATA,3,2 is used after TBTEMP,,CRIT, the criterion used is the inverse of the “strength ratio”:

$$\xi_3 = 1.0 \Big/ \left(-\frac{B}{2A} + \sqrt{(B / 2A)^2 + 1.0 / A}\right) \quad (14.99\text{–}36)$$

where: ξ_3 = value of Tsai–Wu failure criterion (output quantity FC3) (accessed with input **TBDATA** command with **TB**,FAIL)

$$A = -\frac{(\sigma_x)^2}{\sigma^f_{xt}\,\sigma^f_{xc}} - \frac{(\sigma_y)^2}{\sigma^f_{yt}\,\sigma^f_{yc}} - \frac{(\sigma_z)^2}{\sigma^f_{zt}\,\sigma^f_{zc}} + \frac{(\sigma_{xy})^2}{(\sigma^f_{xy})^2} + \frac{(\sigma_{yz})^2}{(\sigma^f_{yz})^2} + \frac{(\sigma_{xz})^2}{(\sigma^f_{xz})^2}$$

$$+ \frac{C_{xy}\sigma_x\sigma_y}{\sqrt{\sigma^f_{xt}\,\sigma^f_{xc}\,\sigma^f_{yt}\,\sigma^f_{yc}}} + \frac{C_{yz}\sigma_y\sigma_z}{\sqrt{\sigma^f_{yt}\,\sigma^f_{yc}\,\sigma^f_{zt}\,\sigma^f_{zc}}} + \frac{C_{xz}\sigma_x\sigma_z}{\sqrt{\sigma^f_{xt}\,\sigma^f_{xc}\,\sigma^f_{zt}\,\sigma^f_{zc}}}$$

$$B = \left(\frac{1}{\sigma^f_{xt}} + \frac{1}{\sigma^f_{xc}}\right)\sigma_x + \left(\frac{1}{\sigma^f_{yt}} + \frac{1}{\sigma^f_{yc}}\right)\sigma_y + \left(\frac{1}{\sigma^f_{zt}} + \frac{1}{\sigma^f_{zc}}\right)\sigma_z$$

C_{xy}, C_{yz}, C_{xz} = x–y, y–z, x–z, respectively, coupling coefficient for Tsai–Wu theory (input on **TBDATA** command with **TB**,FAIL, defaults to –1.0)

The Tsai–Wu failure criteria used here are 3–D versions of the failure criterion reported in of Tsai and Hahn(190) for the 'strength index' and of Tsai(93) for the 'strength ratio'. Apparent differences are:

1. The program input used negative values for compression limits, whereas Tsai uses positive values for all limits.
2. The program uses C_{xy} instead of the F^*_{xy} used by Tsai and Hahn with C_{xy} being twice the value of F^*_{xy}.