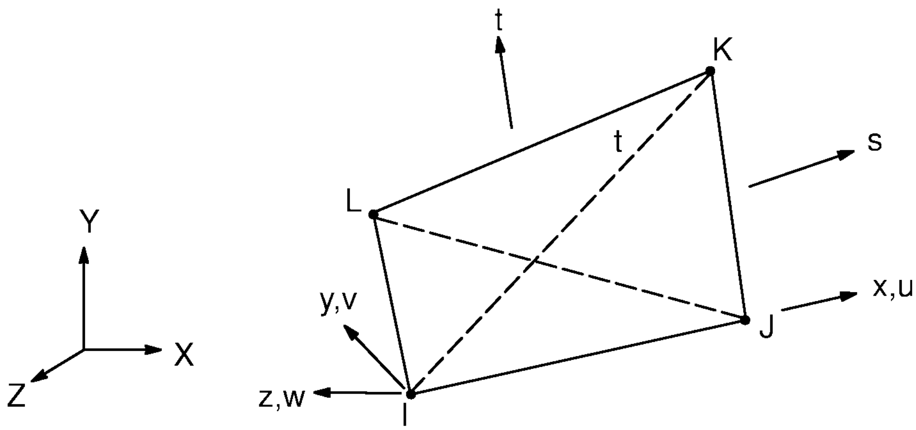
# 14.63 SHELL63 — Elastic Shell




| Matrix or Vector | | Geometry | Shape Functions | Integration Points |
|---|---|---|---|---|
| Stiffness Matrix | Membrane | Quad | Equations (12.5.11–1) and (12.5.11–2) (and, if modified extra shape functions are included (KEYOPT(3)=0) and element has 4 unique nodes, equations (12.5.12–1) thru (12.5.12–3)) | 2 x 2 |
| | | Triangle | Equations (12.5.3–1) thru (12.5.3–3) | 1 |
| | Bending | Four triangles that are overlaid are used. These subtriangles refer to equation (12.5.3–3) | | 3 (for each triangle) |

<table>
<tr><th>Matrix or Vector</th><th></th><th>Geometry</th><th>Shape Functions</th><th>Integration Points</th></tr>
<tr><td rowspan="3">Mass Matrix</td><td rowspan="2">Membrane</td><td>Quad</td><td>Equations (12.5.8–1), (12.5.8–2) and (12.5.8–3)</td><td>2 x 2</td></tr>
<tr><td>Triangle</td><td>Equations (12.5.1–1), (12.5.1–2) and (12.5.1–3)</td><td>1</td></tr>
<tr><td>Bending</td><td colspan="2">Four triangles that are overlaid are used. These triangles connect nodes IJK, IJL, KLI, and KLJ. w is defined as given in Zienkiewicz(39)</td><td>3 (for each triangle)</td></tr>
<tr><td rowspan="2">Transverse Pressure Load Vector</td><td colspan="2">Reduced shell pressure loading (KEYOPT(6)=0) (Load vector excludes moments)</td><td>One–sixth (one–third for triangles) of the total pressure times the area is applied to each node normal of each subtriangle of the element</td><td>None</td></tr>
<tr><td colspan="2">Consistent shell pressure loading (KEYOPT(6)=2) (Load vector includes moments)</td><td>Same as mass matrix</td><td>Same as mass matrix</td></tr>
<tr><td rowspan="2">Edge Pressure Load Vector</td><td>Quad</td><td colspan="2">Equations (12.5.8–1) and (12.5.8–2) specialized to the edge</td><td>2</td></tr>
<tr><td>Triangle</td><td colspan="2">Equations (12.5.1–1) and (12.5.1–2) specialized to the edge</td><td>2</td></tr>
<tr><td>Foundation Stiffness Matrix</td><td colspan="3">Same as mass matrix</td><td>Same as mass matrix</td></tr>
<tr><td>Stress Stiffness Matrix</td><td colspan="3">Same as mass matrix</td><td>Same as mass matrix</td></tr>
<tr><td>Thermal Load Vector</td><td colspan="3">Same as stiffness matrix</td><td>Same as stiffness matrix</td></tr>
</table>

| Load Type | Distribution |
|---|---|
| Element Temperature | Bilinear in plane of element, linear thru thickness |
| Nodal Temperature | Bilinear in plane of element, constant thru thickness |
| Pressure | Bilinear in plane of element, linear along each edge |

## 14.63.1 Other Applicable Sections

Chapter 2 describes the derivation of structural element matrices and load vectors as well as stress evaluations. Section 13.1 describes integration point locations.

## 14.63.2 Foundation Stiffness

If $K_f$, the foundation stiffness, is input, the out–of–plane stiffness matrix is augmented by three or four springs to ground. The number of springs is equal to the number of distinct nodes, and their direction is normal to the plane of the element. The value of each spring is:

$$K_{f,i} = \frac{\Delta \ K_f}{N_d} \qquad (14.63–1)$$

where:
- $K_{f,i}$ = normal stiffness at node i
- $\Delta$ = element area
- $K_f$ = foundation stiffness (input as EFS on **R** command)
- $N_d$ = number of distinct nodes

The output includes the foundation pressure, computed as:

$$\sigma_p = \frac{K_f}{4} \left( w_I + w_J + w_K + w_L \right) \qquad (14.63–2)$$

where:
- $\sigma_p$ = foundation pressure (output as FOUND, PRESS)
- $w_I$, etc. = lateral deflection at node I, etc.

## 14.63.3 In–Plane Rotational Stiffness

The in–plane rotational (drilling) DOF has no stiffness associated with it, based on the shape functions. A small stiffness is added to prevent a numerical instability following the approach presented by Kanok–Nukulchai(26) for non–warped elements if KEYOPT(1) = 0. KEYOPT(3) = 2 is used to include the Allman–type rotational DOFs (as described with SHELL43).

## 14.63.4 Warping

If all four nodes are not defined to be in the same flat plane (or if an initially flat element loses its flatness due to large displacements (**NLGEOM**,ON)), additional calculations are performed in SHELL63. The purpose of the additional calculations is to convert the matrices and load vectors of the element from the points on the flat plane in which the element is derived to the actual nodes. Physically, this may be thought of as adding short rigid offsets between the flat plane of the element and the actual nodes. (For the membrane stiffness only case (KEYOPT(1)=1), the limits given with SHELL41 are used). When these offsets are required, it implies that the element is not flat, but rather it is "warped." To account for the warping, the following procedure is used: First, the normal to element is computed by taking the vector cross–product (the common normal) between the vector from node I to node K and the vector from node J to node L. Then, the check can be made to see if extra calculations are needed to account for warped elements. This check consists of comparing the normal to each of the four element corners with the element normal as defined above. The corner normals are computed by taking the vector cross–product of vectors representing the two adjacent edges. All vectors are normalized to 1.0. If any of the three global Cartesian components of each corner normal differs from the equivalent component of the element normal by more than .00001, then the element is considered to be warped.

A warping factor is computed as:

$$\phi = \frac{D}{t} \tag{14.63–3}$$

where:

$D$ = component of the vector from the first node to the fourth node parallel to the element normal

$t$ = average thickness of the element

If: $\phi \leq$ 0.1 no warning message is printed

$.10 \leq \phi \leq$ 1.0 a warning message is printed

$1.0 < \phi$ a message suggesting the use of triangles is printed and the run terminates

To account for the warping, the following matrix is developed to adjust the output matrices and load vector:

$$[W] = \begin{bmatrix} [w_1] & [0] & [0] & [0] \\ [0] & [w_2] & [0] & [0] \\ [0] & [0] & [w_3] & [0] \\ [0] & [0] & [0] & [w_4] \end{bmatrix} \tag{14.63–4}$$

$$\left[w_i\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & Z_i^o & 0 \\ 0 & 1 & 0 & Z_i^o & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (14.63\text{–}5)$$

where: $Z_i^o$ = offset from the average plane at node i

and the DOF are in the usual order of UX, UY, UZ, ROTX, ROTY, and ROTZ. To ensure the location of the average plane goes through the middle of the element, the following condition is met:

$$Z_1^o + Z_2^o + Z_3^o + Z_4^o = 0 \quad (14.63\text{–}6)$$

## 14.63.5 Options for Non–Uniform Material

SHELL63 can be adjusted for non–uniform materials, using an approach similar to that of Takemoto and Cook(107). Considering effects in the element x direction only, the loads are related to the displacement by:

$$T_x = t\, E_x\, \epsilon_x \quad (14.63\text{–}7)$$

$$M_x = -\frac{t^3\, E_x}{12\left(1 - \nu_{xy}\, \nu_{yx}\right)}\, \kappa_x \quad (14.63\text{–}8)$$

where: $F_x$ = force per unit width

$t$ = thickness (input quantities TK(I), TK(J), TK(K), TK(L) on **R** command)

$E$ = Young’s modulus in x direction (input as EX on **MP** command)

$\varepsilon_x$ = strain of middle fiber in x direction

$M_x$ = moment per unit length

$\nu_{xy}$ = Poisson’s ratio (input as PRXY or NUXY on **MP** command)

$\nu_{yx}$ = Poisson’s ratio (see Section 2.1)

$\kappa_x$ = curvature in x direction

A non–uniform material may be represented with equation (14.63–8) as:

$$M_x = -\, C_r\, \frac{t^3\, E_x}{12\left(1 - \nu_{xy}\, \nu_{yx}\right)}\, \kappa_x \quad (14.63\text{–}9)$$

where: $C_r$ = input quantity RMI on **RMORE** command

The above discussion relates only to the formulation of the stiffness matrix.

Similarly, stresses for uniform materials are determined by:

$$\sigma_x^{top} = E\left(\epsilon_x + \frac{t}{2}\,\kappa_x\right) \qquad (14.63\text{–}10)$$

$$\sigma_x^{bot} = E\left(\epsilon_x - \frac{t}{2}\,\kappa_x\right) \qquad (14.63\text{–}11)$$

where: $\sigma_x^{top}$ = x direction stress at top fiber
$\sigma_x^{bot}$ = x direction stress at bottom fiber

For non–uniform materials, the stresses are determined by:

$$\sigma_x^{top} = E\left(\epsilon_x + c_t\,\kappa_x\right) \qquad (14.63\text{–}12)$$

$$\sigma_x^{bot} = E\left(\epsilon_x - c_b\,\kappa_x\right) \qquad (14.63\text{–}13)$$

where: $c_t$ = input quantity CTOP, **RMORE** command
$c_b$ = input quantity CBOT, **RMORE** command

The output quantities MX, MY, MXY are determined from the output stresses rather than from equation (14.63–9).

## 14.63.6 Usage of ERESX Command

The **ERESX**,NO command may be used to specify that integration point results are to be copied to the nodes. For the case of quadrilateral shaped elements, the bending results of each subtriangle are averaged and copied to the node of the quadrilateral which shares two edges with that subtriangle.