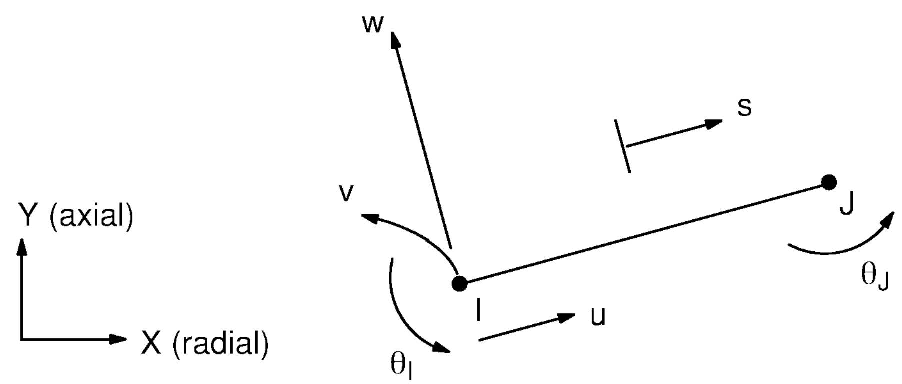
14.61 SHELL61 — Axisymmetric–Harmonic Structural Shell



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equations (12.4.2–1), (12.4.2–2), and (12.4.2–3) . If extra shape functions are not included (KEYOPT(3) = 1): equations (12.4.1–1), (12.4.1–2), and (12.4.1–3)	3 along length
Mass Matrix	Equations (12.3.1–1), (12.3.1–2), and (12.3.1–3)	Same as stiffness matrix
Stress Stiffness Matrix	Equations (12.4.1–1), (12.4.1–2), and (12.4.1–3)	Same as stiffness matrix
Thermal and Pressure Load Vector	Same as stiffness matrix	Same as stiffness matrix

Load Type	Distribution
Element Temperature	Linear through thickness and along length, harmonic around circumference
Nodal Temperature	Constant through thickness, linear along length, harmonic around circumference
Pressure	Linear along length, harmonic around circumference

Reference: Zienkiewicz(39)

14.61.1   Other Applicable Sections

Chapter 2 discusses fundamentals of linear elements. Section 13.1 describes integration point locations. Section 14.25 has a discussion on temperature, applicable to this element.

14.61.2   Assumptions and Restrictions

The material properties are assumed to be constant around the entire circumference, regardless of temperature dependent material properties or loading.

14.61.3   Stress, Force, and Moment Calculations

Element output comes in two forms:

1. Stresses as well as forces and moments per unit length: This printout is controlled by the KEYOPT(6). The thru–the–thickness stress locations are shown in Figure 14.61–1. The stresses are computed using standard procedures as given in Section 2.3. The stresses may then be integrated thru the thickness to give forces per unit length and moments per unit length at requested points along the length:

$$T_x = \sigma_x\big|_c t \qquad (14.61–1)$$

$$T_z = \sigma_z\big|_c t \qquad (14.61–2)$$

$$T_{xz} = \sigma_{xz}\big|_c t \qquad (14.61–3)$$

$$M_x = \left(\sigma_x\big|_t - \sigma_x\big|_b\right)\frac{t^2}{12} \tag{14.61–4}$$

$$M_z = \left(\sigma_z\big|_t - \sigma_z\big|_b\right)\frac{t^2}{12} \tag{14.61–5}$$

$$M_{xz} = \left(\sigma_{xz}\big|_t - \sigma_{xz}\big|_b\right)\frac{t^2}{12} \tag{14.61–6}$$

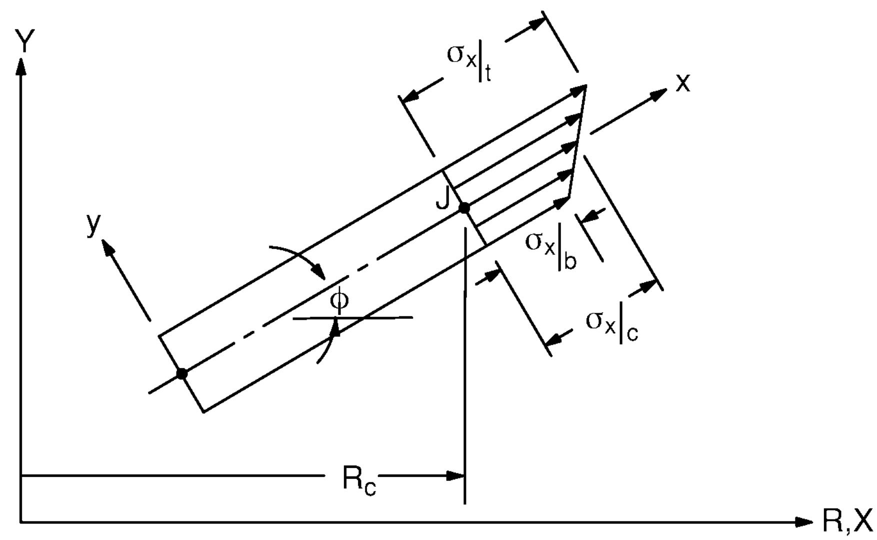


Figure 14.61–1 Stress Locations

where:

T_x, T_z, T_{xz}, M_x, M_z, M_{xz}	=	output as TX, TZ, TXZ, MX, MZ, MXZ, respectively
t	=	thickness (input quantities TK(I), TK(J) on **R** command)
σ_x, σ_y, σ_z, σ_{xz}	=	output as SX, SY, SZ, and SXZ, respectively
$\sigma_x\big|_c$	=	x stress at centerplane (also nodal locations)
	=	$\left(\sigma_x\big|_t + \sigma_x\big|_b\right)/2$
$\sigma_x\big|_t$	=	x stress at top
$\sigma_x\big|_b$	=	x stress at bottom

2. Forces and moments on a circumference basis: This printout is controlled by KEYOPT(4). The values are computed using:

$$\{F_\ell\} = [T_R]^T\left([K_e]\{u_e\} - \{F_e^{th}\} - \{F_e^{pr}\}\right) \tag{14.61–7}$$

where:

$F_\ell = \lfloor F^r_{x,1} \;\; F^r_{y,1} \;\; F^r_{z,1} \;\; M^r_{z,1} \;\; F^r_{x,2} \;\; F^r_{y,2} \;\; F^r_{z,2} \;\; M^r_{z,2} \rfloor^T$
= output as MFOR and MMOM
$[T_R]$ = local to global transformation matrix
$[K_e]$ = element stiffness matrix
$\{u_e\}$ = nodal displacements
$\{F^{th}_c\}$ = element thermal load vector
$\{F^{pr}_c\}$ = element pressure load vector

Another difference between the two types of output are the nomenclature conventions. Since the first group of output uses a shell nomenclature convention and the second group of output uses a nodal nomenclature convention, M_z and M^r_z represent moments in different directions.

The rest of this subsection will describe some of the expected relationships between these two methods of output at the ends of the element. This is done to give a better understanding of the terms, and possibly detect poor internal consistency, suggesting that a finer mesh is in order. It is advised to concentrate on the primary load carrying mechanisms. In order to relate these two types of output in the printout, they have to be requested with both KEYOPT(6) ≥ 1 and KEYOPT(4) = 1. Further, care must be taken to ensure that the same end of the element is being considered.

The axial reaction force based on the stress over an angle $\Delta\beta$ is:

$$F^r_x = \int_{-t/2}^{t/2} \left[\frac{\left(\sigma_x\big|_t + \sigma_x\big|_b\right)}{2} + \frac{\left(\sigma_x\big|_t - \sigma_x\big|_b\right) y}{t} \right] \Delta\beta \, (R_c - y \sin \phi) \, dy \qquad (14.61\text{–}8)$$

or

$$F^r_x = \Delta\beta \left[\frac{\left(\sigma_x\big|_t + \sigma_x\big|_b\right)}{2} R_c t - \left(\sigma_x\big|_t - \sigma_x\big|_b\right) \sin \phi \, \frac{t^2}{12} \right] \qquad (14.61\text{–}9)$$

where:
R_c = radius at midplane
t = thickness

The reaction moment based on the stress over an angle $\Delta\beta$ is:

$$M_x^r = \int_{-t/2}^{t/2} \left[\frac{\left(\sigma_x\big|_t + \sigma_x\big|_b\right)}{2} + \frac{\left(\sigma_x\big|_t - \sigma_x\big|_b\right) y}{t} \right] y \Delta\beta \left(R_c - y \sin \phi\right) dy \qquad (14.61\text{–}10)$$

or

$$M_x^r = \Delta\beta \left[- \frac{\left(\sigma_x\big|_t + \sigma_x\big|_b\right)}{2} \frac{t^3 \sin \phi}{12} + \left(\sigma_x\big|_t - \sigma_x\big|_b\right) R_c \frac{t^2}{12} \right] \qquad (14.61\text{–}11)$$

Since SHELL61 computes stiffness matrices and load vectors using the entire circumference for axisymmetric structures, $\Delta\beta = 2\pi$. Using this fact, the definition of $\sigma_x\big|_c$, and equations (14.61–1) and (14.61–4), equations (14.61–9) and (14.61–11) become:

$$F_x^r = 2\pi \left(R_c T_x - \sin \phi \, M_x\right) \qquad (14.61\text{–}12)$$

$$M_z^r = 2\pi \left(- \frac{t^2 \sin \phi}{12} T_x + R_c M_x \right) \qquad (14.61\text{–}13)$$

As the definition of ϕ is critical for these equations, Figure 14.61–2 is provided to show ϕ in all four quadrants.

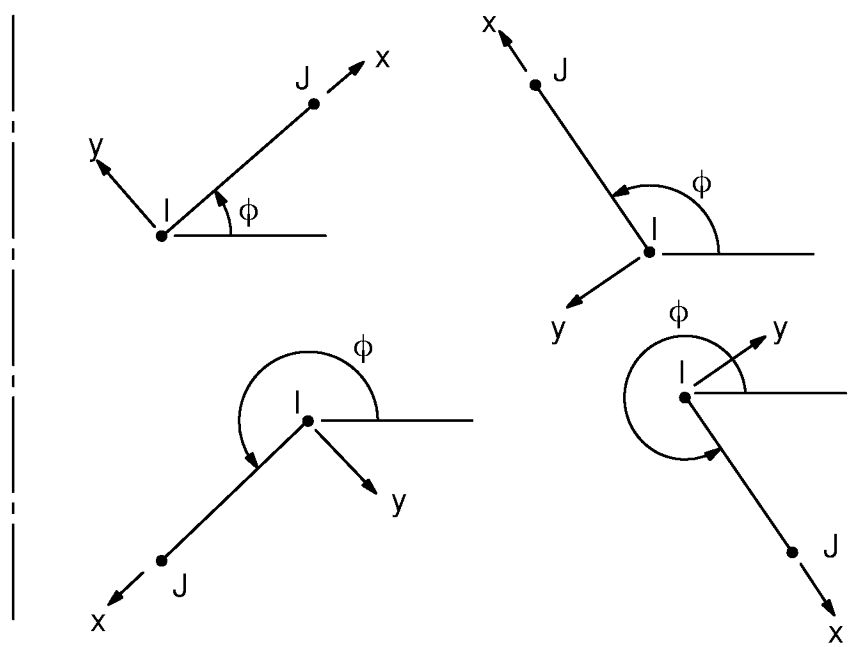


Figure 14.61–2 Element Orientations

In a uniform stress (σ_x) environment, a reaction moment will be generated to account for the greater material on the outside side. This is equivalent to moving the reaction point outward a distance y_f. y_f is computed by:

$$y_f = \frac{M_z^r}{F_x^r} \tag{14.61–14}$$

Using equations (14.61–12) and (14.61–13) and setting M_x to zero gives:

$$y_f = -\frac{t^2 \sin \phi}{12 R_c} \tag{14.61–15}$$