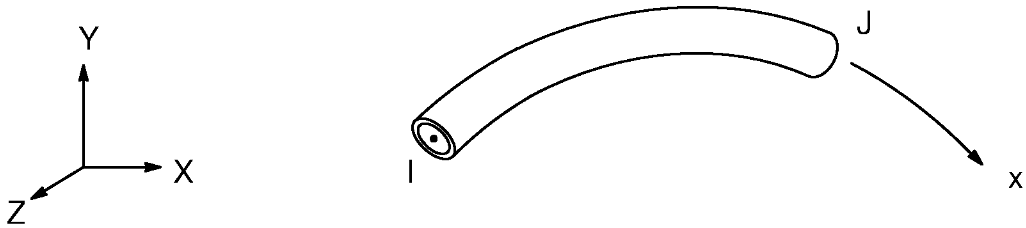
14.60 PIPE60 — Plastic Curved Pipe (Elbow)



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	No shape functions are explicitly used. Rather, a flexibility matrix similar to that developed by Chen (4) is inverted and used.	None
Mass Matrix	No shape functions are used. Rather a lumped mass matrix using only translational DOF is used.	None
Pressure, Thermal, and Newton–Raphson Load Vector	No shape functions are explicitly used. See development below.	8 around circumference at each end of the element. The points are located midway between the inside and outside surfaces

Load Type	Distribution
Element Temperature	Bilinear across cross–section, linear along length
Nodal Temperature	Constant across cross–section, linear along length
Pressure	Internal and External: constant along length and around circumference. Lateral: varies trigonometrically along length

14.60.1 Assumptions and Restrictions

The radius/thickness ratio is assumed to be large.

14.60.2 Other Applicable Sections

The stiffness and mass matrices are identical to those derived for PIPE18 in Section 14.18. Section 14.16 discusses some aspects of the elastic stress printout.

14.60.3 Load Vector

The element load vector is computed in a linear analysis by:

$$\{F_\ell\} = [K_\ell]\left\{u^F\right\} \tag{14.60–1}$$

and in a nonlinear (Newton–Raphson) analysis by:

$$\{F_\ell\} = [K_\ell]\left(\left\{u^F\right\} - \{u_{n-1}\}\right) \tag{14.60–2}$$

where:
- $\{F_\ell\}$ = element load vector (in element coordinates) (applied loads minus Newton–Raphson restoring force) from previous iteration
- $[K_\ell]$ = element stiffness matrix (in element coordinates)
- $\{u^F\}$ = induced nodal displacements in the element (see equation (14.60–3))
- $\{u_{n-1}\}$ = displacements of the previous iteration

The element coordinate system is a cylindrical system as shown in Figure 14.60–1.

The induced nodal displacement vector $\{u^F\}$ is defined by:

$$
\left\{u^F\right\} = \left\{ \begin{array}{ll}
-\frac{R}{4}\sin\frac{\theta}{4}\cos\frac{\theta}{4}\sum_{j=1}^{8}\epsilon_j^{(1)} & \\
0 & \\
-\frac{R}{4}\sin^2\frac{\theta}{4}\sum_{j=1}^{8}\epsilon_j^{(1)} & \\
\frac{R\theta}{4D_m}\sum_{j=1}^{8}\gamma_j^{(1)} & \\
\frac{R\theta}{6D_m}\sum_{j=1}^{8}\frac{\epsilon_j^{(1)}}{\cos\beta_j} & j \neq 2, j \neq 6 \\
\frac{R\theta}{6D_m}\sum_{j=1}^{8}\frac{\epsilon_j^{(1)}}{\sin\beta_j} & j \neq 4, j \neq 8 \\
\frac{R}{4}\sin\frac{\theta}{4}\cos\frac{\theta}{4}\sum_{j=1}^{8}\epsilon_j^{(2)} & \\
0 & \\
-\frac{R}{4}\sin^2\frac{\theta}{4}\sum_{j=1}^{8}\epsilon_j^{(2)} & \\
-\frac{R\theta}{4D_m}\sum_{j=1}^{8}\gamma_j^{(2)} & \\
-\frac{R\theta}{6D_m}\sum_{j=1}^{8}\frac{\epsilon_j^{(2)}}{\cos\beta_j} & j \neq 2, j \neq 6 \\
-\frac{R\theta}{6D_m}\sum_{j=1}^{8}\frac{\epsilon_j^{(2)}}{\sin\beta_j} & j \neq 4, j \neq 8
\end{array} \right\} \tag{14.60–3}
$$

where:

$\epsilon_j^{(1)} = \epsilon^{th} + \epsilon_x^{pr} + \epsilon_x^{pl} + \epsilon_x^{cr} + \epsilon^{sw}$ at end I

$\epsilon_j^{(2)} = \epsilon^{th} + \epsilon_x^{pr} + \epsilon_x^{pl} + \epsilon_x^{cr} + \epsilon^{sw}$ at end J

$\gamma_j^{(1)} = \gamma_{xh}^{pl} + \gamma_{xh}^{cr}$ at end I

$\gamma_j^{(2)} = \gamma_{xh}^{pl} + \gamma_{xh}^{cr}$ at end J

ϵ^{th} = $\alpha \left(T_j - T_{REF}\right)$ (= thermal strain)

α = thermal coefficient of expansion (input as ALPX on **MP** command)

T_j = temperature at integration point j

ϵ_x^{pr} = axial strain due to pressure (see equation (14.16–10)

ϵ_x^{pl} = plastic axial strain (see Section 4.1)

ϵ_x^{cr} = axial creep strain (see Section 4.3)

ϵ^{sw} = swelling strain (see Section 4.4)

γ_{xh}^{pl} = plastic shear strain (see Section 4.1)

γ_{xh}^{cr} = creep shear strain (see Section 4.3)

R = radius of curvature (input as RADCUR on **R** command)

D_m = 1/2 (D_o + D_i) (= average diameter)

D_o = outside diameter (input as OD on **R** command)

D_i = D_o – 2t (= inside diameter)

t = thickness (input as TKWALL on **R** command)

θ = subtended angle of the elbow

β_j = angular position of integration point j on the circumference Figure 14.60–2 (output quantity ANGLE)

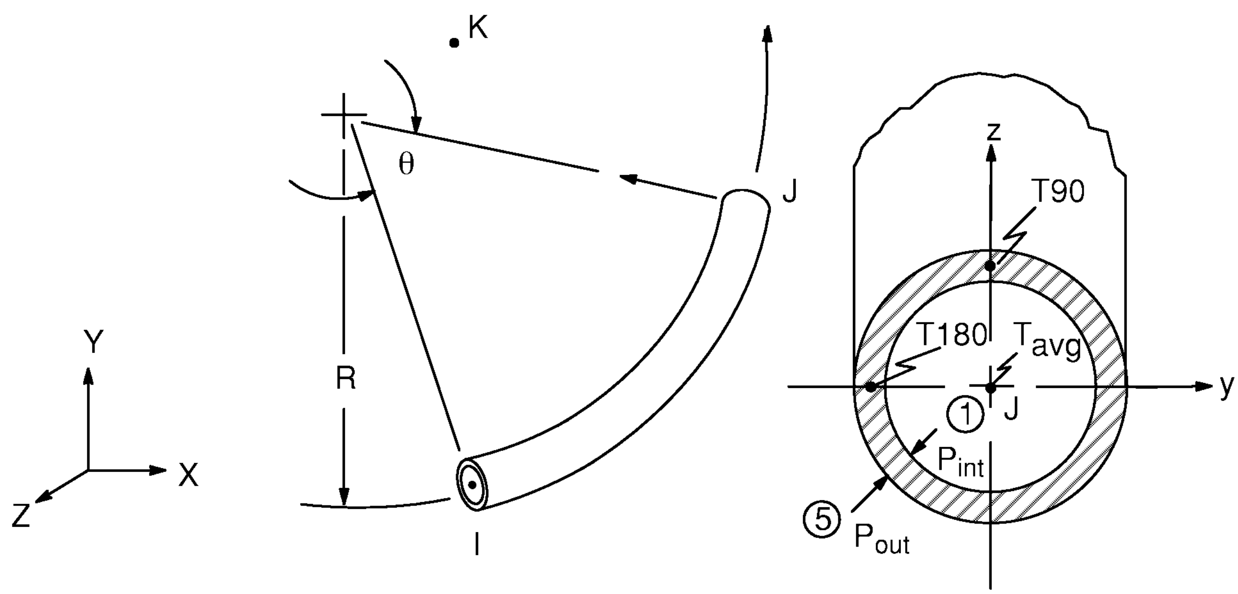


Figure 14.60–1 3–D Plastic Curved Pipe Element Geometry

There are eight integration points around the circumference at each end of the element, as shown in Figure 14.60–2. The assumption has been made that the elbow has a large radius–to–thickness ratio so that the integration points are located at the midsurface of the shell. Since there are integration points only at each end of the element, the subtended angle of the element should not be too large. For example, if there are effects other than internal pressure and in–plane bending, the elements should have a subtended angle no larger than 45°.

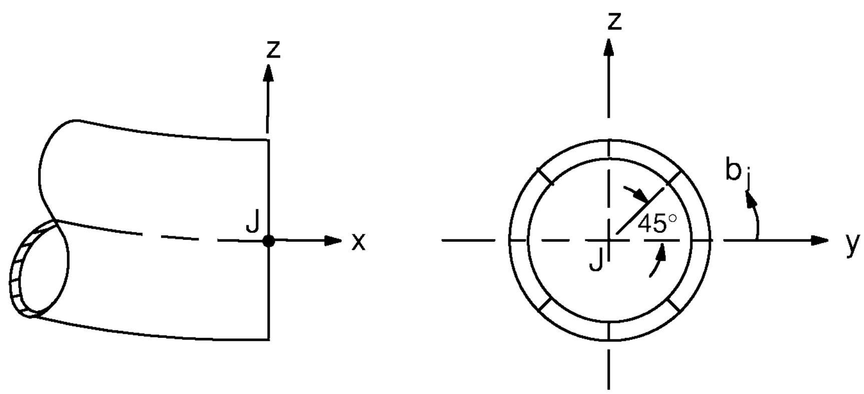


Figure 14.60–2 Integration Point Locations at End J

14.60.4 Stress Calculations

The stress calculations take place at each integration point, and have a different basis than for PIPE18, the elastic elbow element. The calculations have three phases:

1. Computing the modified total strains (ε').
2. Using the modified total strains and the material properties, computing the change in plastic strains and then the current elastic strains.
3. Computing the current stresses based on the current elastic strains.

Phase 2 is discussed in Section 4.1. Phase 1 and 3 are discussed below. Phase 1: The modified total strains at an integration point are computed as:

$$\{\epsilon'\} = [D]^{-1}\{\sigma_b\} \tag{14.60–4}$$

where: $$\epsilon' = \begin{Bmatrix} \epsilon_x^{d'} \\ \epsilon_h^{d'} \\ \epsilon_r' \\ \gamma_{xh}' \end{Bmatrix}$$

$$[D]^{-1} = \begin{bmatrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 \\ 0 & 0 & 0 & \frac{2(1+\nu)}{E} \end{bmatrix}$$

x, h, r = subscripts representing axial, hoop, and radial directions, respectively

E = Young's modulus (input as EX on **MP** command)

ν = Poisson's ratio (input as PRXY or NUXY on **MP** command)

$\{\sigma_b\}$, the integration point stress vector before plasticity computations, is defined as:

$$\{\sigma_b\} = \begin{Bmatrix} \sigma_x \\ \sigma_h \\ \sigma_r \\ \tau_{xh} \end{Bmatrix} \tag{14.60–5}$$

These terms are defined by:

$$\sigma_x = \frac{F_x}{A^w} + S_y M_y + S_z M_z + \frac{D_i P_i - D_o P_o}{4t} \tag{14.60–6}$$

$$\sigma_h = \nu S_y M_y + \nu S_z M_z + \left(\frac{D_o}{2t} - \frac{2}{5} \right) \left[\frac{R + \frac{1}{2} r \sin\phi_j}{R + r \sin\phi_j} \right] (P_i - P_o) \tag{14.60–7}$$

$$\sigma_r = -\frac{P_i + P_o}{2} \tag{14.60–8}$$

$$\tau_{xh} = -\frac{2}{A^w} \left(F_y \cos\beta_j + F_z \sin\beta_j \right) - \frac{S_x M_x}{2} \tag{14.60–9}$$

where: F_y, F_z, M_x = forces on element at node by integration point (see equation (14.60–10) below)

A^w = $\frac{\pi}{4} \left(D_o^2 - D_i^2 \right)$

S_x = $\frac{32 D_o}{\pi \left(D_o^4 - D_i^4 \right)}$

S_y = $- S_x \left(\sin\phi_j + C_2 \left((1.5 C_1 - 18.75) \sin 3\phi_j + 11.25 \sin 5\phi_j \right) \right)$

$$S_z = S_x\left(\cos\phi_j + C_2\left((1.5C_1 - 18.75)\cos3\phi_j + 11.25\cos5\phi_j\right)\right)$$

$$\phi_j = \beta_j - \frac{\pi}{2}$$

$$r = \frac{D_o + D_i}{4}$$

P_i = internal pressure (input on **SFE** command)

P_o = external pressure (input on **SFE** command)

$$C_1 = 17 + 600\,C_3^{\,2} + 480\,\frac{PR^2}{Ert}$$

$$C_2 = \frac{1}{\left(1 - \nu^2\right)\left(C_1C_4 - 6.25 - 4.5C_1\right)}$$

$$C_3 = \frac{Rt}{r^2\,\sqrt{1 - \nu^2}}$$

$$C_4 = 5 + 6\,C_3^{\,2} + 24\,\frac{PR^2}{Ert}$$

$$P = P_i - P_o$$

Note that S_y and S_z are expressed in three–term Fourier series around the circumference of the pipe cross–section. These terms have been developed from the ASME Code(60). Note also that ϕ_j is the same angle from the element y axis as β_j is for PIPE20. The forces on both ends of the element (F_y, M_x, etc.) are computed from:

$$\{F_e\} = [T_R]\left(\left[K_c^p\right]\{\Delta u_e\} - \{F_\ell\}\right) \qquad (14.60–10)$$

where:

$\{F_e\}$ = $\lfloor F_{xI}\ \ldots\ M_{zJ}\rfloor^T$ = forces on element in element coordinate system

$[T_R]$ = global to local conversion matrix (note that the local x axis is not straight but rather is curved along the centerline of the element)

$[K_e]$ = element stiffness matrix (global Cartesian coordinates)

$\{\Delta u_e\}$ = element incremental displacement vector

Phase 3: Performed after the plasticity calculations, Phase 3 is done simply by:

$$\{\sigma\} = [D]\{\epsilon^c\} \qquad (14.60–11)$$

where:

$\{\varepsilon^e\}$ = elastic strain after the plasticity calculations

The $\{\sigma\}$ vector, which is used for output, is defined with the same terms as in equation (14.60–5). But lastly, σ_r is redefined by equation (14.60–8) as this stress value must be maintained, regardless of the amount of plastic strain.

As long as the element remains elastic, additional printout is given during the solution phase. The stress intensification factors (C_σ) of PIPE18 are used in this printout, but are not used in the printout associated with the plastic stresses and strains. The maximum principal stresses, the stress intensity, and equivalent stresses are compared (and replaced if necessary) to the values of the plastic printout at the eight positions around the circumference at each end. Also, the elastic printout is based on outer–fiber stresses, but the plastic printout is based on mid–thickness stresses. Further, other thin–walled approximations in equations (14.60–6) and (14.60–7) are not used by the elastic printout. Hence some apparent inconsistency appears in the printout.