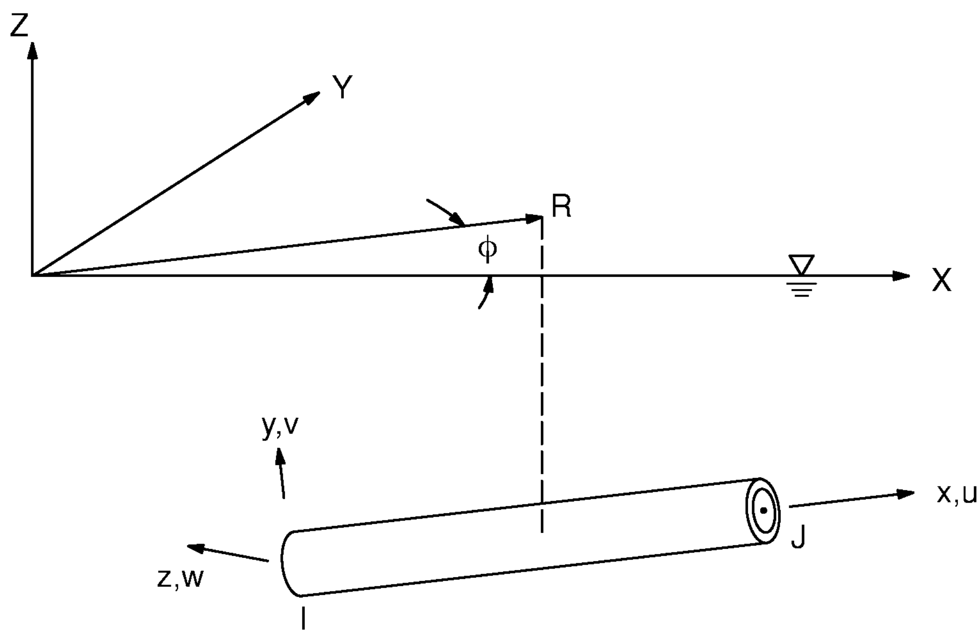
14.59 PIPE59 — Immersed Pipe or Cable



Matrix or Vector	Options	Shape Functions	Integration Points
Stiffness Matrix	Pipe Option (KEYOPT(1) ≠ 1)	Equations (12.2.2–1), (12.2.2–2), (12.2.2–3), and (12.2.2–4)	None
	Cable Option (KEYOPT(1) = 1)	Equation (12.2.1–1), (12.2.1–2), and (12.2.1–3)	None
Stress Stiffness Matrix	Pipe Option (KEYOPT(1) ≠ 1)	Equation (12.2.2–2) and (12.2.2–3)	None
	Cable Option (KEYOPT(1) = 1)	Equation (12.2.1–2) and (12.2.1–3)	None

Matrix or Vector	Options	Shape Functions	Integration Points
Mass Matrix	Pipe Option (KEYOPT(1)$\neq$1) with consistent mass matrix (KEYOPT(2)=0)	Equation (12.2.2–1), (12.2.2–2), and (12.2.2–3)	None
	Cable Option (KEYOPT(1)=1) or reduced mass matrix (KEYOPT(2)=1)	Equation (12.2.1–1), (12.2.1–2), and (12.2.1–3)	None
Thermal, Pressure, and Hydrostatic Load Vector	Same as stiffness matrix		None
Hydrodynamic Load Vector	Same as stiffness matrix		2

Load Type	Distribution
Element Temperature*	Linear thru thickness or across diameter, and along length
Nodal Temperature*	Constant across cross–section, linear along length
Pressure	Linearly varying (in Z direction) internal and external pressure caused by hydrostatic effects. Exponentially varying external overpressure (in Z direction) caused by hydrodynamic effects

* Immersed elements with no internal diameter assume the temperatures of the water.

14.59.1 Overview of the Element

PIPE59 is similar to PIPE16 (or LINK8 if the cable option (KEYOPT(1)=1) is selected). The principal differences are that the mass matrix includes the:

1. Outside mass of the fluid (“added mass”) (acts only normal to the axis of the element),
2. Internal structural components (pipe option only),

and the load vector includes:

a. Hydrostatic effects

b. Hydrodynamic effects

14.59.2 Location of the Element

The origin for any problem containing PIPE59 must be at the free surface (mean sea level). Further, the Z axis is always the vertical axis, pointing away from the center of the earth.

The element may be located in the fluid, above the fluid, or in both regimes simultaneously. There is a tolerance of only $\frac{D_e}{8}$ below the mud line, for which

$$D_e = D_o + 2\ t_i \qquad (14.59\text{–}1)$$

where:

t_i = thickness of external insulation (input quantity TKIN on **RMORE** command)

D_o = outside diameter of pipe/cable (input quantity DO on **R** command)

The mud line is located at distance d below the origin, where d is the input quantity DEPTH on the **TBDATA** commands with **TB**,WATER (water motion table). This condition is checked with:

$$Z(N) > -\left(d + \frac{D_e}{8}\right) \quad \leftarrow \text{no error message} \qquad (14.59\text{–}2)$$

$$Z(N) \leq -\left(d + \frac{D_e}{8}\right) \quad \leftarrow \text{fatal error message} \qquad (14.59\text{–}3)$$

where Z(N) is the vertical location of node N. If it is desired to generate a structure below the mud line, the user can set up a second material property for those elements using a greater d and deleting hydrodynamic effects. Alternatively, the user can use a second element type such as PIPE16, the elastic straight pipe element.

If the problem is a large deflection problem, greater tolerances apply for second and subsequent iterations:

$$Z(N) > -(d + 10\ D_e) \quad \leftarrow \text{no error message} \qquad (14.59\text{–}4)$$

$$-(d + 10\ D_e) \geq Z(N) > -(2d) \quad \leftarrow \text{warning message} \qquad (14.59\text{–}5)$$

$$-\,(2d)\ \geq\ Z(N) \qquad \leftarrow \text{fatal error message} \qquad (14.59\text{–}6)$$

where Z(N) is the present vertical location of node N. In other words, the element is allowed to sink into the mud for 10 diameters before generating a warning message. If a node sinks into the mud a distance equal to the water depth, the run is terminated. If the element is supposed to lie on the ocean floor, gap elements must be provided.

14.59.3 Stiffness Matrix

The element stiffness matrix for the pipe option (KEYOPT(1)≠ 1) is the same as for BEAM4 (equation (14.4–1)), except that $K_\ell(4,1) = K_\ell(1,4) = K_\ell(10,7) = K_\ell(7,10) = T_T$ and $K_\ell(7,4) = K_\ell(4,7) = K_\ell(10,1) = K_\ell(1,10) = -T_T$.

where:

$$T_T = \begin{cases} 0 & \text{if KEYOPT(1) = 0 (standard option for torque balanced cable or pipe)} \\ \dfrac{G_T\left(D_o^3 - D_i^3\right)}{L} & \text{if KEYOPT(1) = 2 (twist–tension option for non–torque balanced cable or pipe)} \end{cases}$$

G_T = twist–tension stiffness constant, which is a function of the helical winding of the armoring (input quantity GXZ on **MP** command, may be negative)

D_i = inside diameter of pipe = $D_o - 2\,t_w$

t_w = wall thickness (input quantity TWALL on **R** command)

L = element length

$A = \frac{\pi}{4}\left(D_o^2 - D_i^2\right)$ = cross–sectional area

$I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)$ = moment of inertia

J = 2I

The element stiffness matrix for the cable option (KEYOPT(1) = 1) is the same as for LINK8.

14.59.4 Mass Matrix

The element mass matrix for the pipe option (KEYOPT(1) ≠ 1) and KEYOPT(2) = 0) is the same as for BEAM4 (equation (14.4–2)), except that $M_\ell(1,1)$, $M_\ell(7,7)$, $M_\ell(1,7)$, and $M_\ell(7,1)$, as well as M(4,4), M(10,10), M(4,10), and M(10,4), are multiplied by the factor (M_a / M_t).

where: $M_t = (m_w + m_{int} + m_{ins} + m_{add})\,L$

= mass/unit length for motion normal to the axis of the element

M_a = $(m_w + m_{int} + m_{ins})\,L$
= mass/unit length for motion parallel to the axis of the element

m_w = $(1-\varepsilon^{in})\varrho\frac{\pi}{4}\left(D_o^2 - D_i^2\right)$

ϱ = density of the pipe wall (input as DENS on **MP** command)

ε^{in} = initial strain (input quantity ISTR on **RMORE** command)

m_{int} = mass/unit length of the internal fluid and additional hardware (input quantity CENMPL on **RMORE** command)

m_{ins} = $(1-\varepsilon_{in})\ \varrho_i\ \frac{\pi}{4}\left(D_e^2 - D_o^2\right)$

ϱ_i = density of external insulation (input quantity DENSIN on **RMORE** command)

m_{add} = $(1-\varepsilon_{in})\ C_I\varrho_w\ \frac{\pi}{4}\ D_e^2$

C_I = coefficient of added mass of the external fluid (input quantity CI on **RMORE** command)

ϱ_w = fluid density (input quantity DENSW on **TBDATA** commands with **TB**,WATER)

The element mass matrix for the cable option (KEYOPT(1) = 1) or the reduced mass matrix option (KEYOPT(2) $\neq$ 0) is the same form as for LINK8 except that $M_\ell(1,1)$, $M_\ell(4,4)$, $M_\ell(1,4)$ and $M_\ell(4,1)$ are also multiplied by the factor (M_a/M_t).

14.59.5 Load Vector

The element load vector consists of two parts:

1. Distributed force per unit length to account for hydrostatic (buoyancy) effects ($\{F/L\}_b$) as well as axial nodal forces due to internal pressure and temperature effects $\{F_x\}$.
2. Distributed force per unit length to account for hydrodynamic effects (current and waves) ($\{F/L\}_d$).

The hydrostatic and hydrodynamic effects work with the original diameter and length, i.e., initial strain and large deflection effects are not considered.

Hydrostatic Effects

Hydrostatic effects may affect the outside and the inside of the pipe. Pressure on the outside crushes the pipe and buoyant forces on the outside tend to raise the pipe to the water surface. Pressure on the inside tends to stabilize the pipe cross–section.

The buoyant force for a totally submerged element acting in the positive z direction is:

$$\{F/L\}_b = C_b\rho_w\,\frac{\pi}{4}\,D_e^2\ \{g\} \tag{14.59–7}$$

where: $\{F/L\}_b$ = vector of loads per unit length due to buoyancy
C_b = coefficient of buoyancy (input quantity CB on **RMORE** command)
$\{g\}$ = acceleration vector

Also, an adjustment for the added mass term is made.

The crushing pressure at a node is:

$$P_o^s = -\rho_w gz + P_o^a \tag{14.59–8}$$

where: P_o^s = crushing pressure due to hydrostatic effects
g = acceleration due to gravity
z = vertical coordinate of the node
P_o^a = input external pressure (input on **SFE** command)

The internal (bursting) pressure is:

$$P_i = -\rho_o g(z - S_{fo}) + P_i^a \tag{14.59–9}$$

where: P_i = internal pressure
ϱ_o = internal fluid density (input quantity DENSO on **R** command)
S_{fo} = z coordinate of free surface of fluid (input quantity FSO on **R** command)
P_i^a = input internal pressure (input quantity on **SFE** command)

To ensure that the problem is physically possible as input, a check is made to see if the cross–section collapses under the hydrostatic effects. The cross–section is assumed to be unstable if:

$$P_o^s - P_i > \frac{E}{4(1 - \nu^2)} \left(\frac{2t_w}{D_o}\right)^3 \tag{14.59–10}$$

where: E = Young's modulus (input as EX on **MP** command)
ν = Poisson's ratio (input as PRXY or NUXY on **MP** command)

The axial force correction term (F_x) is computed as

$$F_x = AE\epsilon_x \tag{14.59–11}$$

where ε_x, the axial strain (see equation 2.1–12) is:

$$\epsilon_x = \alpha\Delta T + \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_h + \sigma_r\right)\right) \tag{14.59–12}$$

where: α = coefficient of thermal expansion (input as ALPX on **MP** command)

ΔT = $T_a - T_{REF}$

T_a = average element temperature

T_{REF} = input on **TREF** command

σ_x = axial stress, computed below

σ_h = hoop stress, computed below

σ_r = radial stress, computed below

The axial stress, assuming the ends are closed, is:

$$\sigma_x = \frac{P_i D_i^2 - P_o D_o^2}{D_o^2 - D_i^2} \qquad (14.59\text{–}13)$$

and using the Lamé stress distribution,

$$\sigma_h = \frac{P_i D_i^2 - P_o D_o^2 + \frac{D_i^2 D_o^2}{D^2}\left(P_i - P_o\right)}{D_o^2 - D_i^2} \qquad (14.59\text{–}14)$$

$$\sigma_r = \frac{P_i D_i^2 - P_o D_o^2 - \frac{D_i^2 D_o^2}{D^2}\left(P_i - P_o\right)}{D_o^2 - D_i^2} \qquad (14.59\text{–}15)$$

where: P_o = $P_o^s + P_o^d$

P_o^d = hydrodynamic pressure, described below

D = diameter being studied

P_i and P_o are taken as average values along each element. Combining equations (14.59–12) thru (14.59–15).

$$\epsilon_x = \alpha\Delta T + \frac{1 - 2\nu}{E}\,\frac{P_i D_i^2 - P_o D_o^2}{D_o^2 - D_i^2} \qquad (14.59\text{–}16)$$

Note that if the cross–section is solid (D_i = 0.), equation (14.59–14) reduces to:

$$\epsilon_x = \alpha\Delta T - \frac{1 - 2\nu}{E}\,P_o \qquad (14.59\text{–}17)$$

Hydrodynamic Effects

Hydrodynamic effects may occur because the structure moves in a motionless fluid, the structure is fixed but there is fluid motion, or both the structure and fluid are moving. The fluid motion consists of two parts: current and wave motions. The current is input by giving the current velocity and direction (input quantities W(i) and θ(i)) at up to eight different vertical stations (input quantity Z(i)). The velocity and direction are interpolated linearly between stations. The current is assumed to flow horizontally only. The wave may be input using one of four wave theories in Table 14.59–1 with the input coming from the **TBDATA** commands with **TB**,WATER.

Table 14.59–1 Wave Theory Table

KWAV	Description of Wave Theory
0	Small amplitude wave theory, modified with empirical depth decay function, (Wheeler(35))
1	Small amplitude wave theory, unmodified (Airy wave theory), (Wheeler(35))
2	Stokes fifth order wave theory, (Skjelbreia et al(31))
3	Stream function wave theory, (Dean(59))

The free surface of the wave is defined by

$$\eta_s = \sum_{i=1}^{N_w} \eta_i = \sum_{i=1}^{N_w} \frac{H_i}{2} \cos \beta_i \qquad (14.59\text{–}18)$$

where:
η_s = total wave height

N_w = number of wave components = $\begin{cases} \text{number of waves} & \text{if } K_w \neq 2 \\ 5 & \text{if } K_w = 2 \end{cases}$

K_w = wave theory key (input quantity KWAV on the **TBDATA** commands with **TB**,WATER)

η_i = wave height of component i

H_i = surface coefficient = $\begin{cases} \text{input quantity A(i)} & \text{if } K_w = 0 \text{ or } 1 \\ \text{derived from other input} & \text{if } K_w = 2 \text{ or } 3 \end{cases}$

$$\beta_i = \begin{cases} 2\pi\left(\frac{R}{\lambda_i} + \frac{t}{\tau_i} + \frac{\psi_i}{360}\right) & \text{if KEYOPT(5) = 0 and } K_w = 0 \text{ or } 1 \\ 2\pi\left(\frac{R}{\lambda_i} + \frac{t}{\tau_i} + \frac{\psi_i}{360}\right)(i) & \text{if KEYOPT(5) = 0 and } K_w = 2 \text{ or } 3 \\ 0.0 & \text{if KEYOPT(5) = 1} \\ \frac{\pi}{2} & \text{if KEYOPT(5) = 2} \\ -\frac{\pi}{2} & \text{if KEYOPT(5) = 3} \\ \pi & \text{if KEYOPT(5) = 4} \end{cases}$$

R = radial distance to point on element from origin in the X–Y plane in the direction of the wave

λ_i = wave length = { input quantity WL(i) if WL(i) > 0.0 and if K_W = 0 or 1; otherwise derived from equation (14.59–19)

t = time elapsed (input quantity TIME on **TIME** command) (Note that the default value of TIME is usually not desired. If zero is desired, 10^{-12} can be used).

τ_i = wave period = { input quantity τ(i) if $K_W \neq 3$; derived from other input if K_W +3

ψ_i = phase shift = input quantity ψ(i)

If WL(i) is not input (set to zero) and $K_W < 2$, λ_i is computed iteratively from:

$$\lambda_i = \lambda_i^d \tanh\left(\frac{2\pi d}{\lambda_i}\right) \tag{14.59–19}$$

where:

λ_i = output quantity small amplitude wave length

λ_i^d = $\frac{g(\tau_i)^2}{2\pi}$ = output quantity deep water wave length

g = acceleration due to gravity (Z direction) (input on **ACEL** command)

d = water depth (input quantity DEPTH on the **TBDATA** commands with **TB**,WATER)

Each component of wave height is checked that it satisfies the "Miche criterion" if $K_W \neq 3$. This is to ensure that the wave is not a breaking wave, which the included wave theories do not cover. A breaking wave is one that spills over its crest, normally in shallow water. A warning message is issued if:

$$H_i > H_b \tag{14.59–20}$$

where: $H_b = 0.142\,\lambda_i \tanh\left(\frac{2\pi d}{\lambda_i}\right)$ = height of breaking wave (14.59–21)

When using wave loading, there is an error check to ensure that the input acceleration does not change after the first load step, as this would imply a change in the wave behavior between load steps.

For K_w = 0 or 1, the particle velocities at integration points are computed as a function of depth from:

$$\vec{v}_R = \sum_{i=1}^{N_w} \frac{\cosh\left(k_i \bar{Z} f\right)}{\sinh\left(k_i d\right)} \frac{2\pi}{\tau_i} \eta_i + \vec{v}_D \tag{14.59–22}$$

$$\vec{v}_Z = \sum_{i=1}^{N_w} \frac{\sinh\left(k_i \bar{Z} f\right)}{\sinh\left(k_i d\right)} \dot{\eta}_i \tag{14.59–23}$$

where:

$\vec{v}_R$ = radial particle velocity

$\vec{v}_Z$ = vertical particle velocity

k_i = $2\pi/\lambda_i$

$\bar{Z}$ = height of integration point above the ocean floor = d+Z

$\dot{\eta}_i$ = time derivative of η_i

$\vec{v}_D$ = drift velocity (input quantity W on **TBDATA** command with **TB**,WATER)

$$f = \begin{cases} \dfrac{d}{d + \eta_s} & \text{if } K_w = 0 \text{ (Wheeler(35))} \\ 1.0 & \text{if } K_w = 1 \text{ (small amplitude wave theory)} \end{cases}$$

The particle accelerations are computed by differentiating $\vec{v}_R$ and $\vec{v}_Z$ with respect to time. Thus:

$$\dot{\vec{v}}_R = \sum_{i=1}^{N_w} \frac{\cosh\left(k_i \bar{Z} f\right)}{\sinh\left(k_i d\right)} \left(\frac{2\pi}{\tau_i}\right) \left(\dot{\eta}_i - C\eta_i\right) \tag{14.59–24}$$

$$\dot{\vec{v}}_Z = \sum_{i=1}^{N_w} \frac{\sinh\left(k_i \bar{Z} f\right)}{\sinh\left(k_i d\right)} \left(\frac{2\pi}{\tau_i}\right) \left(-\frac{2\pi}{\tau_i} \eta_i - C\eta_i \left(\frac{\tau}{2\pi}\right)\right) \tag{14.59–25}$$

where: $$C = \begin{cases} \dot{\eta}_s \dfrac{2\pi}{\lambda_i} \dfrac{\overline{Z}\, d}{(d + \eta_s)^2} & \text{if } K_W = 0 \text{ (Wheeler (35))} \\ 0.0 & \text{if } K_W = 1 \text{ (small amplitude wave theory)} \end{cases}$$

Expanding equation 2.29 of the Shore Protection Manual (43) for a multiple component wave, the wave hydrodynamic pressure is:

$$P_o^d = \rho_w\, g \sum_{i=1}^{N_w} \eta_i \frac{\cosh\left[2\pi \frac{\overline{Z}}{\lambda_i}\right]}{\cosh\left[2\pi \frac{d}{\lambda_i}\right]} \tag{14.59–26}$$

However, use of this equation leads to non–zero total pressure at the surface at the crest or trough of the wave. Thus, equation (14.59–26) is modified to be:

$$P_o^d = \rho_w\, g \sum_{i=1}^{N_w} \eta_i \frac{\cosh\left[2\pi \frac{\overline{Z} d}{\lambda_i\,(d + \eta_s)}\right]}{\cosh\left[2\pi \frac{d}{\lambda_i}\right]} \tag{14.59–27}$$

which does result in a total pressure of zero at all points of the free surface. This dynamic pressure, which is calculated at the integration points during the stiffness pass, is extrapolated to the nodes for the stress pass. The hydrodynamic pressure for Stokes fifth order wave theory is:

$$P_o^d = \rho_w\, g \sum_{i=1}^{5} \eta_i \frac{\cosh\left(2\pi \frac{\overline{Z}}{\lambda_i}\right)}{\cosh\left(2\pi \frac{d}{\lambda_i}\right)} \tag{14.59–28}$$

Other aspects of the Stokes fifth order wave theory are discussed by Skjelbreia et al (31). The modification as suggested by Nishimura et al (143) has been included. The stream function wave theory is described by Dean(59).

If both waves and current are present, the question of wave–current interaction must be dealt with. Three options are made available thru K_{cr} (input quantity KCRC on the **TBDATA** commands with **TB**,WATER):

For K_{cr} = 0, the current velocity at all points above the mean sea level is simply set equal to W_o, where W_o is the input current velocity at Z = 0.0. All points below the mean sea level have velocities selected as though there were no wave.

For K_{cr} = 1, the current velocity profile is “stretched” or “compressed” to fit the wave. In equation form, the Z coordinate location of current measurement is adjusted by

$$Z'(j) = Z(j) \frac{d + \eta_s}{d} \qquad (14.59\text{–}29)$$

where: Z(j) = Z coordinate location of current measurement (input quantity Z(j))

Z(j) = adjusted value of Z(j)

For K_{cr} = 2, the same adjustment as for K_{cr} = 1 is used, as well as a second change that accounts for “continuity.” That is,

$$W'(j) = W(j) \frac{d}{d + \eta_s} \qquad (14.59\text{–}30)$$

where: W(j) = velocity of current at this location (input quantity W(j))

W(j) = adjusted value of W(j)

These three options are shown pictorially in Figure 14.59–1.

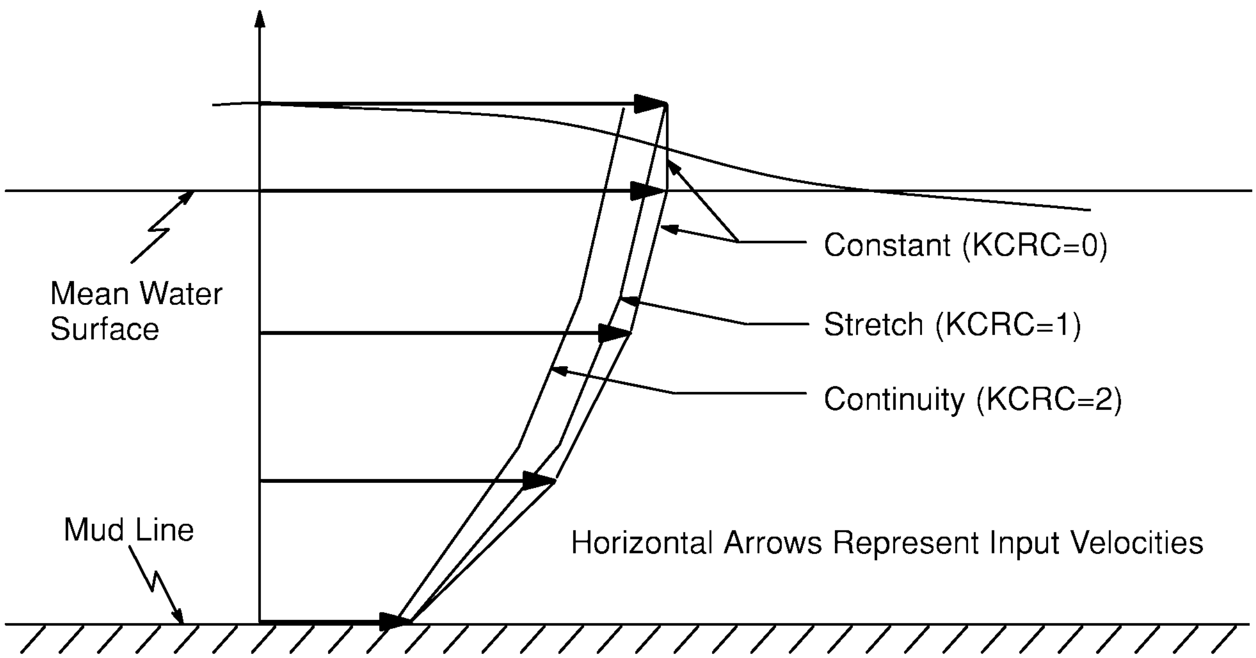


Figure 14.59–1 Velocity Profiles for Wave–Current Interactions

To compute the relative velocities ($\{\dot{u}_n\}$, $\{\dot{u}_t\}$), both the fluid particle velocity and the structure velocity must be available so that one can be subtracted from the other. The fluid particle velocity is computed using relationships such as equations (14.59–22) and (14.59–23) as well as current effects. The structure velocity is available through the Newmark time integration logic (see Section 17.2).

Finally, a generalized Morison's equation is used to compute a distributed load on the element to account for the hydrodynamic effects:

$$\{F/L\}_d = C_D \rho_w \frac{D_e}{2} \left|\{\dot{u}_n\}\right| \{\dot{u}_n\} + C_M \rho_w \frac{\pi}{4} D_e^2 \{\dot{v}_n\} + C_T \rho_w \frac{D_e}{2} \left|\{\dot{u}_t\}\right| \{\dot{u}_t\} \qquad (14.59\text{–}31)$$

where:
- $\{F/L\}_d$ = vector of loads per unit length due to hydrodynamic effects
- C_D = coefficient of normal drag (see below)
- ρ_w = water density $\left(\frac{\text{mass}}{\text{length}^3}\right)$ (input quantity DENSW on **TBDATA** command with **TB**,WATER)
- D_e = outside diameter of the pipe with insulation (length)
- $\{\dot{u}_n\}$ = normal relative particle velocity vector $\left(\frac{\text{length}}{\text{time}}\right)$
- C_M = coefficient of inertia (input quantity CM on **R** command)
- $\{\dot{v}_n\}$ = normal particle acceleration vector $\left(\frac{\text{length}}{\text{time}^2}\right)$
- C_T = coefficient of tangential drag (see below)
- $\{\dot{u}_t\}$ = tangential relative particle velocity vector $\left(\frac{\text{length}}{\text{time}}\right)$

Two integration points along the length of the element are used to generate the load vector. Integration points below the mud line are simply bypassed. For elements intersecting the free surface, the integration points are distributed along the wet length only. If the reduced load vector option is requested (KEYOPT(2) = 2), the moment terms are set equal to zero.

The coefficients of drag (C_D,C_T) may be defined in one of two ways:

1. They may be input as fixed numbers using the real constant table (input quantities CD and CT on **R** and **RMORE** commands), or
2. They may be input as functions of Reynold's number using the water motion table (input quantities RE, CD, and CT on the **TBDATA** commands with **TB**,WATER).

The dependency on Reynold's number (Re) may be expressed as:

$$C_D = f_D\,(Re) \qquad (14.59\text{–}32)$$

where:
- f_D = relationship defined by input quantities RE and CD of the water motion table

Re = $\{\dot{u}_n\}\dfrac{D_e\rho_w}{\mu}$

μ = viscosity (input as VISC on **MP** command)

and

$$C_T = f_T\ (Re) \qquad (14.59–33)$$

where: f_T = relationships defined by input quantities RE and CT on the **TBDATA** commands with **TB**,WATER

Re = $\{\dot{u}_t\}\dfrac{D_e\rho_w}{\mu}$

μ may be input as a temperature–dependent quantity, where the temperatures used are those given by input quantities T(i) of the water motion table.

14.59.6 Stress Output

The below two equations are specialized either to end I or to end J.

The stress output for the pipe format (KEYOPT(1) $\neq$ 1), is similar to PIPE16 (Section 14.16). The average axial stress is:

$$\sigma_x = \frac{F_n}{A} + \frac{D_i^2\ P_i\ -\ D_o^2\ P_o}{D_o^2\ -\ D_i^2} \qquad (14.59–34)$$

where: σ_x = average axial stress (output quantity SAXL)

F_n = axial element reaction force (output quantity FX, adjusted for sign)

P_i = internal pressure

P_o = external pressure = $P_o^s + P_o^d$

and the hoop stress is:

$$\sigma_h = \frac{2\ P_i\ D_i^2 -\ P_o\left(D_o^2 + D_i^2\right)}{D_o^2\ -\ D_i^2} \qquad (14.59–35)$$

where: σ_h = hoop stress at the outside surface of the pipe (output quantity SH)

Average values of P_i and P_o are reported as first and fifth items of the output quantities ELEMENT PRESSURES. Equation (14.59–35) is a specialization of equation (14.59–14). The outside surface is chosen as the bending stresses usually dominate over pressure induced stresses.

All stress results are given at the nodes of the element. However, the hydrodynamic pressure had been computed only at the two integration points. These two values are then used to compute hydrodynamic pressures at the two nodes of the element by extrapolation.

The stress output for the cable format (KEYOPT(1) = 1 with D_i = 0.0) is similar to that for LINK8 (Section 14.8), except that the stress is given with and without the external pressure applied:

$$\sigma_{xI} = \frac{F_\ell}{A} + P_o \tag{14.59–36}$$

$$\sigma_{eI} = \frac{F_\ell}{A} \tag{14.59–37}$$

$$F_a = A\sigma_{xI} \tag{14.59–38}$$

where:
σ_{xI} = output quantity SAXL
σ_{eI} = output quantity SEQV
F_ℓ = axial force on node (output quantity FX)
F_a = axial force in the element (output quantity FAXL)