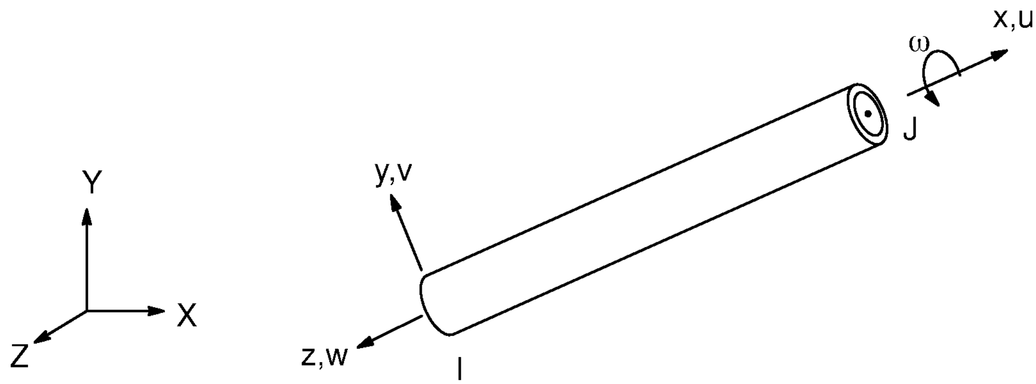
14.20 PIPE20 — Plastic Straight Pipe



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equation (12.2.2–1), (12.2.2–2), (12.2.2–3), and (12.2.2–4)	None for elastic matrix. Same as Newton–Raphson load vector for tangent matrix with plasticity
Stress Stiffness Matrix	Equations (12.2.2–2) and (12.2.2–3)	None
Mass Matrix	Same as stiffness matrix	None
Pressure and Thermal Load Vector	Equations (12.2.2–1), (12.2.2–2), and (12.2.2–3)	None
Newton–Raphson Load Vector	Same as stiffness matrix	2 along the length and 8 points around circumference. The points are located midway between the inside and outside surfaces.

Load Type	Distribution
Element Temperature	Linear across diameter and along length
Nodal Temperature	Constant across cross–section, linear along length
Pressure	Internal and External: constant along length and around circumference Lateral: constant along length

14.20.1 Assumptions and Restrictions

The radius/thickness ratio is assumed to be large.

14.20.2 Other Applicable Sections

Section 14.4 has an elastic beam element stiffness and mass matrix explicitly written out. Section 14.16 discusses the effect of element pressure and the elastic stress printout. Section 14.23 defines the tangent matrix with plasticity and the Newton–Raphson load vector.

14.20.3 Stress and Strain Calculation

PIPE20 uses four components of stress and strain in the stress calculation:

$$\{\sigma\} = \begin{Bmatrix} \sigma_x \\ \sigma_h \\ \sigma_r \\ \sigma_{xh} \end{Bmatrix} \tag{14.20–1}$$

where x, h, r are subscripts representing the axial, hoop and radial directions, respectively. Since only the axial and shear strains can be computed directly from the strain–displacement matrices, the strains are computed from the stresses as follows.

The stresses (before plasticity adjustment) are defined as:

$$\sigma_x = E\epsilon' + \frac{\pi}{4A^w}\left(D_i^{\,2}P_i - D_o^{\,2}P_o\right) \tag{14.20–2}$$

$$\sigma_h = \frac{1}{2t}\left(D_iP_i - D_oP_o\right) \tag{14.20–3}$$

$$\sigma_r = -\frac{1}{2}\left(P_i - P_o\right) \tag{14.20–4}$$

$$\sigma_{xh} = \frac{2}{A^w}\left(F_y \sin\beta_j - F_z \cos\beta_j\right) + \frac{M_x D_m}{2 J} \tag{14.20–5}$$

where:
- ϵ' = modified axial strain (see Section 14.23)
- E = Young's modulus (input as EX on **MP** command)
- P_i = internal pressure (input on **SFE** command)
- P_o = external pressure (input on **SFE** command)
- D_i = internal diameter = $D_o - 2t$
- D_o = input as OUTER DIA. on **R** command
- t = wall thickness (input as WALL THICKNESS on **R** command)
- A^w = $\frac{\pi}{4}\left(D_o^2 - D_i^2\right)$ = wall area
- J = $\frac{\pi}{4} D_m^3 t$
- D_m = $\left(D_i + D_o\right)/2$ = average diameter
- β_j = angular position of integration point J (see Figure 14.20–1) (output quantity ANGLE)
- F_y, F_z, M_x = forces on element node by integration point

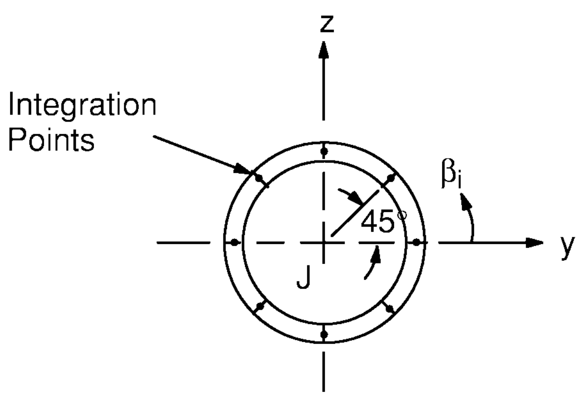


Figure 14.20–1 Integration Points for End J

The forces on the element (F_y, F_z, M_x) are computed from:

$$\{F_\ell\} = [T_R]\left([K_e]\{\Delta u_e\} - \{F_e\}\right) \tag{14.20–6}$$

where:
- $\{F_\ell\}$ = output quantities FORCES ON MEMBER AT NODE
- $[T_R]$ = global to local conversion matrix
- $[K_e]$ = element stiffness matrix

$\{\Delta u_e\}$ = element incremental displacement vector
$\{F_e\}$ = element load vector from pressure, thermal and Newton–Raphson restoring force effects

The forces $\{F_\ell\}$ are in element coordinates while the other terms are given in global Cartesian coordinates. The forces used in equation (14.20–5) correspond to either those at node I or node J, depending at which end the stresses are being evaluated.

The modified total strains for the axial and shear components are readily calculated by:

$$\epsilon_x' = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_h + \sigma_r\right)\right) \tag{14.20–7}$$

$$\epsilon_{xh}' = \frac{\sigma_{xh}}{G} \tag{14.20–8}$$

where: ν = Poisson's ratio (input as PRXY or NUXY on **MP** command)
G = shear modulus (input as GXY on **MP** command)

The hoop and radial modified total strains are computed through:

$$\epsilon_h' = \epsilon_{h,n-1} + \Delta\epsilon_h \tag{14.20–9}$$

$$\epsilon_r' = \epsilon_{r,n-1} + \Delta\epsilon_r \tag{14.20–10}$$

where: $\varepsilon_{h,n-1}$ = hoop strain from the previous iteration
$\varepsilon_{r,n-1}$ = radial strain from the previous iteration
$\Delta\varepsilon_h$ = increment in hoop strain
$\Delta\varepsilon_r$ = increment in radial strain

The strains from the previous iterations are computed using:

$$\epsilon_{h,n-1} = \frac{1}{E}\left(\sigma_h - \nu\left(\sigma_{x,n-1} + \sigma_r\right)\right) \tag{14.20–11}$$

$$\epsilon_{r,n-1} = \frac{1}{E}\left(\sigma_r - \nu\left(\sigma_{x,n-1} + \sigma_h\right)\right) \tag{14.20–12}$$

where $\sigma_{x,n-1}$ is computed using equation (14.20–2) with the modified total strain from the previous iteration. The strain increments in equations (14.20–9) and (14.20–10) are computed from the strain increment in the axial direction:

$$\Delta\epsilon_h = \overline{D}_n^h \Delta\epsilon_x \tag{14.20–13}$$

$$\Delta\epsilon_r = \overline{D}^r_n \Delta\epsilon_x \tag{14.20–14}$$

where:

$\Delta\varepsilon_x$ = $\epsilon' - \epsilon'_{n-1}$ = axial strain increment

$\overline{D}^h_n$, $\overline{D}^r_n$ = factors relating axial strain increment to hoop and radial strain increments, respectively

These factors are obtained from the static condensation of the 3–D elasto–plastic stress–strain matrix to the 1–D component, which is done to form the tangent stiffness matrix for plasticity.

Equations (14.20–7) through (14.20–10) define the four components of the modified total strain from which the plastic strain increment vector can be computed (see Section 4.1). The elastic strains are:

$$\left\{\epsilon^{el}\right\} = \left\{\epsilon'\right\} - \left\{\Delta\epsilon^{pl}\right\} \tag{14.20–15}$$

where:

$\{\varepsilon^{el}\}$ = output quantities EPELAXL, EPELRAD, EPELH, EPELXH

$\{\Delta\varepsilon^{pl}\}$ = plastic strain increment

The stresses are then:

$$\{\sigma\} = [D] \left\{\epsilon^{el}\right\} \tag{14.20–16}$$

where:

$\{\sigma\}$ = output quantities SAXL, SRAD, SH, SXH

$[D]$ = elastic stress–strain matrix

The definition of $\{\sigma\}$ given by equation (14.20–16) is modified in that σ_h and σ_r are redefined by equations (14.20–3) and (14.20–4) as the stress values and must be maintained, regardless of the amount of plastic strain.

As long as the element remains elastic, additional printout is given during the solution phase. The stress intensification factors (C_σ) of PIPE16 are used in this printout, but are not used in the printout associated with the plastic stresses and strains. The maximum principal stresses, the stress intensity, and equivalent stresses are compared (and replaced if necessary) to the values of the plastic printout at the eight positions around the circumference at each end. Also, the elastic printout is based on stresses at the outer fiber, but the plastic printout is based on mid–thickness stresses. Hence, some apparent inconsistency appears in the printout.