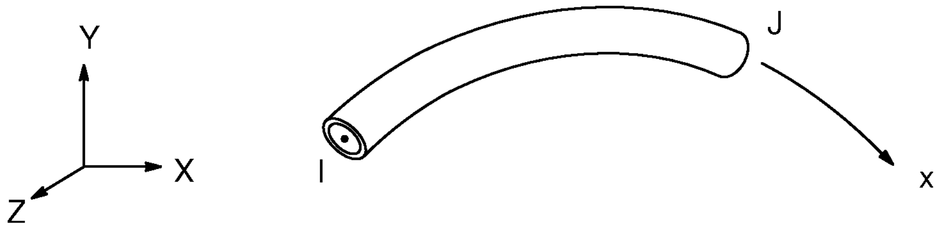
14.18 PIPE18 — Elastic Curved Pipe (Elbow)



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	No shape functions are explicitly used. Rather a flexibility matrix similar to that developed by Chen(4) is inverted and used.	None
Mass Matrix	No shape functions are used. Rather a lumped mass matrix using only translational degrees of freedom is used.	None
Thermal and Pressure Load Vector	Equation (12.2.2–1), (12.2.2–2), and (12.2.2–3)	None

Load Type	Distribution
Element Temperature	Linear thru thickness or across diameter, and along length
Nodal Temperature	Constant across cross–section, linear along length
Pressure	Internal and External: constant along length and around the circumference Lateral: varies trigonometrically along length (see below)

14.18.1 Other Applicable Sections

Section 14.16 covers some of the applicable stress calculations.

14.18.2 Stiffness Matrix

The geometry in the plane of the element is given in Figure 14.18–1.

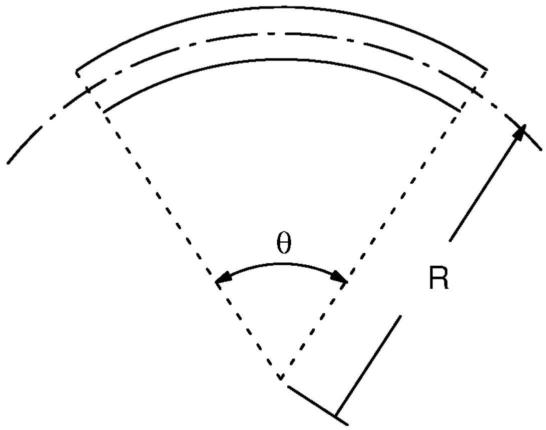


Figure 14.18–1 Plane Element

The stiffness matrix is developed based on an approach similar to that of Chen(4). The flexibility of one end with respect to the other is:

$$[f] = \begin{bmatrix} f_{11} & 0 & f_{13} & 0 & f_{15} & 0 \\ 0 & f_{22} & 0 & f_{24} & 0 & f_{26} \\ f_{31} & 0 & f_{33} & 0 & f_{35} & 0 \\ 0 & f_{42} & 0 & f_{44} & 0 & f_{46} \\ f_{51} & 0 & f_{53} & 0 & f_{55} & 0 \\ 0 & f_{62} & 0 & f_{64} & 0 & f_{66} \end{bmatrix} \qquad (14.18\text{–}1)$$

where:

$$f_{11} = \frac{R^3 C_{fi}}{EI}\left(\frac{\theta}{2}\cos\theta - \frac{3}{2}\sin\theta + \theta\right) + \frac{R}{2EA^w}(\theta\cos\theta + \sin\theta) + \frac{2R(1+\nu)}{EA^w}(\theta\cos\theta - \sin\theta)$$

$$f_{13} = -f_{31} = \frac{R^3 C_{fi}}{EI}\left(\cos\theta - 1 + \frac{\theta}{2}\sin\theta\right) + \frac{R\theta\sin\theta}{EA^w}\left(\frac{5}{2} + 2\nu\right)$$

$$f_{15} = f_{51} = \frac{R^2 C_{fi}}{EI} (\sin\theta - \theta)$$

$$f_{22} = \frac{R^3(1+\nu)}{EI}(\theta - \sin\theta) + \frac{R^3}{2EI}\left(1+\nu+C_{fo}\right)(\theta\cos\theta - \sin\theta) + \frac{R\theta\,(4(1+\nu))}{EA^w}$$

$$f_{24} = f_{42} = \frac{R^2}{2EI}\left(1+\nu+C_{fo}\right)(\theta\cos\theta - \sin\theta)$$

$$f_{26} = -f_{62} = \frac{R^2}{EI}\left((1+\nu)(\cos\theta - 1) + \frac{\theta}{2}\sin\theta\,(1+\nu+C_{fo})\right)$$

$$f_{33} = \left(\frac{\theta}{2}\cos\theta - \frac{1}{2}\sin\theta\right)\left(\frac{R^3C_{fi}}{EI} + \frac{R}{EA^w}\right) + \left(\frac{\theta}{2}\cos\theta + \frac{1}{2}\sin\theta\right)\left(\frac{4R(1+\nu)}{EA^w}\right)$$

$$f_{35} = -f_{53} = \frac{R^2C_{fi}}{EI}(\cos\theta - 1)$$

$$f_{44} = \frac{R}{2EI}\left(1+\nu+C_{fo}\right)\theta\cos\theta + \frac{R}{2EI}\left(1+\nu-C_{fo}\right)\sin\theta$$

$$f_{46} = -f_{64} = \frac{R}{2EI}\left(1+\nu+C_{fo}\right)\theta\sin\theta$$

$$f_{55} = \frac{RC_{fi}}{EI}\theta$$

$$f_{66} = \frac{R}{2EI}\left(\left(1+\nu+C_{fo}\right)\theta\cos\theta - \left(1+\nu-C_{fo}\right)\sin\theta\right)$$

and where:

R = radius of curvature (input as RADCUR on **R** command) (see Figure 14.18–1)

θ = included angle of element (see Figure 14.18–1)

E = Young's modulus (input as EX on **MP** command)

ν = Poisson's ratio (input as PRXY or NUXY on **MP** command)

I = moment of inertia of cross–section = $\frac{\pi}{64}\left(D_o^4 - D_i^4\right)$

A^w = area of cross–section = $\frac{\pi}{4}\left(D_o^2 - D_i^2\right)$

D_o = outside diameter (input as OD on **R** command)

D_i = $D_o - 2t$ = inside diameter

t = wall thickness (input as TKWALL on **R** command)

$$C_{fi} = \begin{cases} C'_{fi} & \text{if } C'_{fi} > 0.0 \\ \dfrac{1.65}{h} \text{ or } 1.0, & \text{whichever is greater if } C'_{fi} = 0.0 \text{ and KEYOPT(3)=0 (ASME flexibility factor, ASME Code(40))} \\ \dfrac{1.65}{h\left(1 + \dfrac{PrX_K}{tE}\right)} \text{ or } 1.0 & \text{whichever is greater if } C'_{fi} = 0.0 \text{ and KEYOPT(3)=1 (ASME flexibility factor, ASME Code(40))} \\ \dfrac{10 + 12h^2}{1 + 12h^2} & \text{if } C'_{fi} = 0.0 \text{ and KEYOPT(3) = 2 (Karman flexiblity factor)} \end{cases}$$

C'_{fi} = input as FLXI on **R** command

$h = \dfrac{tR}{r^2}$

r = average radius $\dfrac{(D_o - t)}{2}$

$$P = \begin{cases} P_i - P_o & \text{if } P_i - P_o > 0.0 \\ 0.0 & \text{if } P_i - P_o \leq 0.0 \end{cases}$$

P_i = internal pressure (input on **SFE** command)

P_o = external pressure (input on **SFE** command)

$$X_K = \begin{cases} 6\left(\dfrac{r}{t}\right)^{\frac{4}{3}}\left(\dfrac{R}{r}\right)^{\frac{1}{3}} & \text{if } \dfrac{R}{r} \geq 1.7 \\ 0.0 & \text{if } \dfrac{R}{r} < 1.7 \end{cases}$$

$$C'_{fo} = \begin{cases} C'_{fo} \text{ if } C'_{fo} > 0.0 \\ C_{fi} \text{ if } C'_{fo} = 0.0 \end{cases}$$

C'_{fo} = input as FLXO on **RMORE** command

The user should not use the KEYOPT(3) = 1 option if:

$$\theta_c R < 2r \qquad (14.18\text{–}2)$$

where: θ_c = included angle of the complete elbow, not just the included angle for this element (θ)

Next, the 6 x 6 stiffness matrix is derived from the flexibility matrix by inversion:

$$[K_o] = [f]^{-1} \qquad (14.18\text{–}3)$$

The full 12 x 12 stiffness matrix (in element coordinates) is derived by expanding the 6 x 6 matrix derived above and transforming to the global coordinate system.

14.18.3 Mass Matrix

The element mass matrix is a diagonal (lumped) matrix with each translation term being defined as:

$$m_t = \frac{m_e}{2} \tag{14.18–4}$$

where:
- m_t = mass at each node in each translation direction
- m_e = total mass of element
  = $(\varrho A^w + \varrho_{fl} A^{fl} + \varrho_{in} A^{in}) R\theta$
- ϱ = pipe wall density (input as DENS on **MP** command)
- ϱ_{fl} = internal fluid density (input as DENSFL on **RMORE** command)
- A^{fl} = $\frac{\pi}{4} D_i^{\,2}$
- ϱ_{in} = insulation density (input as DENSIN on **RMORE** command)
- A^{in} = insulation cross–section area
  = $\frac{\pi}{4}\left(D_{o+}^2 - D_o^{\,2}\right)$
- D_{o+} = $D_o + 2\,t^{in}$
- t^{in} = insulation thickness (input as TKIN on **RMORE** command)

14.18.4 Load Vector

The load vector in element coordinates due to thermal and pressure effects is:

$$\left\{F_\ell^{th}\right\} + \left\{F_\ell^{pr,i}\right\} = R\, \epsilon_x\, [K_e]\, \{A\} + \{F_\ell^{pr,t}\} \tag{14.18–5}$$

where:
- ε_x = strain caused by thermal as well as internal and external pressure effects (see equation (14.16–10))
- $[K_e]$ = element stiffness matrix in global coordinates
- $\{A\}$ = $\lfloor 0\ 0\ 1\ 0\ 0\ 0 \quad 0\ 0\ 1\ 0\ 0\ 0 \rfloor^T$
- $\left\{F_\ell^{pr,t}\right\}$ = element load vector due to transverse pressure

$\left\{F_\ell^{pr,t}\right\}$ is computed based on input quantities PX, PY, PZ and curved beam formulas from Roark(48). Table 18, reference no. (loading) 3, 4, and 5 and 5c was used for in–plane effects and Table 19, reference no. (end restraint) 4e was used for out–of–plane effects. As a radial load varying trigonometrically along the length of the element was not one of the available cases given in Roark(48), an integration of a point radial load was done, using Loading 5c.

14.18.5 Stress Calculations

In the stress pass, the stress evaluation is similar to that for PIPE16 (Section 14.16). It is not the same as for PIPE60. The wall thickness is diminished by t_c, the corrosion allowance (input as TKCORR on **R** command). See also Section 14.16. The bending stress components are multiplied by stress intensification factors (C_σ). The "intensified" stresses are used in the principal and combined stress calculations. The factors are:

$$C_{\sigma,I} = \begin{cases} C_o & \text{, if SIFI} < 1.0 \\ \text{input as SIFI on } \mathbf{R} \text{ command, if SIFI} > 1.0 \end{cases} \quad (14.18\text{–}6)$$

$$C_{\sigma,J} = \begin{cases} C_o & \text{, if SIFJ} < 1.0 \\ \text{input as SIFJ on } \mathbf{R} \text{ command, if SIFJ} > 1.0 \end{cases} \quad (14.18\text{–}7)$$

$$C_o = \begin{cases} \dfrac{0.9}{h_e^{\;2/3}} & \text{whichever is greater (ASME Code (40))} \\ 1.0 & \end{cases} \quad (14.18\text{–}8)$$

where:

$$h_e = \frac{16\ t_e\ R}{\left(D_i + d_o\right)^2}$$

$$t_e = t - t_c$$

$$d_o = D_o - 2\ t_c$$