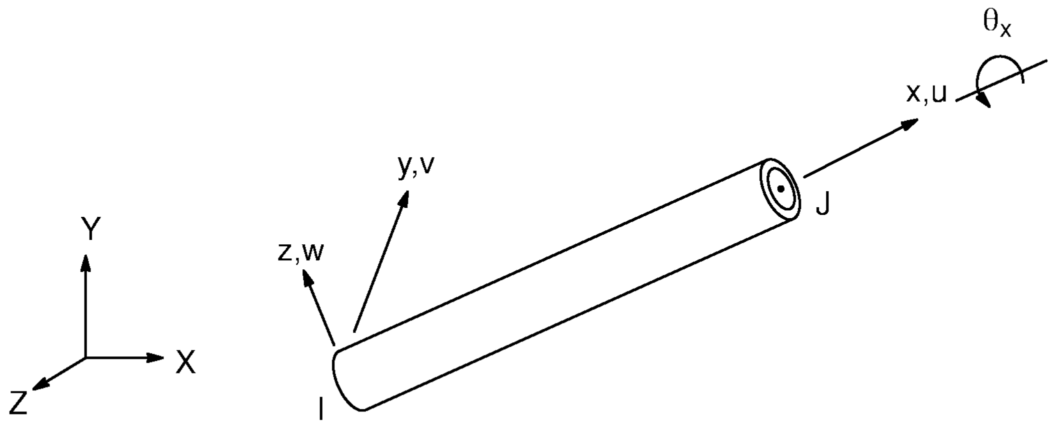
14.16 PIPE16 — Elastic Straight Pipe



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equation (12.2.2–1), (12.2.2–2), (12.2.2–3), and (12.2.2–4)	None
Mass Matrix	Same as stiffness matrix	None
Stress Stiffness Matrix	Equation (12.2.2–2) and (12.2.2–3)	None
Load Vector (Pressure and Thermal)	Equation (12.2.2–1), (12.2.2–2), and (12.2.2–3)	None

Load Type	Distribution
Element Temperature	Linear thru thickness or across diameter, and along length
Nodal Temperature	Constant across cross–section, linear along length
Pressure	Internal and External: constant along length and around circumference. Lateral: constant along length

14.16.1 Other Applicable Sections

The basic form of the element matrices is given with the 3–D beam element, BEAM4 (Section 14.4).

14.16.2 Assumptions and Restrictions

The element is assumed to be a thin–walled pipe except as noted. The corrosion allowance is used only in the stress evaluation, not in the matrix formulation.

14.16.3 Stiffness Matrix

The element stiffness matrix of PIPE16 is the same as for BEAM4, except that

$$A = A^w = \frac{\pi}{4}\left(D_o^2 - D_i^2\right) = \text{pipe wall cross–sectional area} \tag{14.16–1}$$

$$I_y = I_z = I = \frac{\pi}{64}\left(D_o^4 - D_i^4\right)\frac{1}{C_f} = \text{bending moment of inertia} \tag{14.16–2}$$

$$J = \frac{\pi}{32}\left(D_o^4 - D_i^4\right) = \text{torsional moment of inertia} \tag{14.16–3}$$

and,

$$A_{si} = \frac{A}{2.0} = \text{shear area} \tag{14.16–4}$$

where:
π = 3.141592653
D_o = outside diameter (input quantity OD on **R** command)
D_i = inside diameter = $D_o - 2t_w$
t_w = wall thickness (input quantity TKWALL on **R** command)
$C_f = \begin{cases} 1.0 & \text{if } f = 0.0 \\ f & \text{if } f > 0.0 \end{cases}$
f = flexibility factor (input quantity FLEX on **R** command)

Further, the axial stiffness of the element is defined as

$$K_\ell(1,1) = \begin{cases} \dfrac{A^w E}{L} & \text{if } k = 0.0 \\ k & \text{if } k > 0.0 \end{cases} \tag{14.16–5}$$

where: $K_\ell(1,1)$ = axial stiffness of the element

E = Young's modulus (input as EX on **MP** command)

L = element length

k = alternate axial pipe stiffness (input quantity STIFF on **RMORE** command)

14.16.4 Mass Matrix

The element mass matrix of PIPE16 is the same as for BEAM4, except total mass of the element is assumed to be:

$$m_e = m_e^w + \left(\rho_{fl} A^{fl} + \rho_{in} A^{in}\right) L \qquad (14.16\text{–}6)$$

where: m_e = total mass of element

m_e^w = pipe wall mass

$$= \begin{cases} \rho A^w L & \text{if } m_w = 0.0 \\ m_w & \text{if } m_w > 0.0 \end{cases}$$

m_w = alternate pipe wall mass (input quantity MWALL on **RMORE** command)

ϱ = pipe wall density (input as DENS on **MP** command)

ϱ_{fl} = internal fluid density (input quantity DENSFL on **R** command)

A^{fl} = $\frac{\pi}{4} D_i^{\,2}$

ϱ_{in} = insulation density (input quantity DENSIN on **RMORE** command)

A^{in} = insulation cross–sectional area

$$= \begin{cases} \frac{\pi}{4}\left(D_{o+}^2 - D_o^2\right) & \text{if } A_s^{in} = 0.0 \\ \frac{A^{in}\, t^{in}}{L} & \text{if } A_s^{in} > 0.0 \end{cases}$$

D_{o+} = $D_o + 2t^{in}$

t^{in} = insulation thickness (input quantity TKIN on **RMORE** command)

A_s^{in} = alternate representation of the surface area of the outside of the pipe element (input quantity AREAIN on **RMORE** command)

Also, the bending moments of inertia (equation (14.16–2)) are used without the C_f term.

14.16.5 Gyroscopic Damping Matrix

The element gyroscopic damping matrix is:

$$[C_e] = 2\Omega\rho AL \begin{bmatrix} 0 & & & & & & & & & & & \\ 0 & 0 & & & & & & & & & & \\ 0 & -g & 0 & & & & & & & & & \\ 0 & 0 & 0 & 0 & & & & & \text{Antisymmetric} & & & \\ 0 & -h & 0 & 0 & 0 & & & & & & & \\ 0 & 0 & -h & 0 & -i & 0 & & & & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & & & & & \\ 0 & 0 & -g & 0 & -h & 0 & 0 & 0 & & & & \\ 0 & g & 0 & 0 & 0 & -h & 0 & -g & 0 & & & \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & & \\ 0 & -h & 0 & 0 & 0 & j & 0 & h & 0 & 0 & 0 & \\ 0 & 0 & -h & 0 & -j & 0 & 0 & 0 & h & 0 & -i & 0 \end{bmatrix} \quad (14.16\text{–}7)$$

where:

Ω = rotation frequency about the positive x axis (input quantity SPIN on **RMORE** command)

$g = \dfrac{6/5\ r^2}{L^2\,(1+\phi)^2}$

$h = \dfrac{-\,(1/10 - 1/2\ \phi)\ r^2}{L\,(1+\phi)^2}$

$i = \dfrac{(2/15 + 1/6\ \phi + 1/3\ \phi^2)\ r^2}{(1+\phi)^2}$

$j = \dfrac{-\,(1/30 + 1/6\ \phi - 1/6\ \phi^2)\ r^2}{(1+\phi)^2}$

$r = \sqrt{I/A}$

$\phi = \dfrac{12\ EI}{G\ A^s L^2}$

G = shear modulus (input as GXY on **MP** command)

A^s = shear area ($=A^w/2.0$)

14.16.6 Stress Stiffness Matrix

The element stress stiffness matrix of PIPE16 is identical to that for BEAM4.

14.16.7 Load Vector

The element pressure load vector is

$$\{F_\ell\} = \begin{Bmatrix} F_1 \\ F_2 \\ \vdots \\ F_{12} \end{Bmatrix} \qquad (14.16\text{–}8)$$

where:

$F_1 = F_A + F_P$

$F_7 = -F_A + F_P$

$F_A = A^w \, E \, \epsilon_x^{pr}$

ϵ_x^{pr} = axial strain due to pressure load, defined below

$$F_P = \begin{cases} 0.0 & \text{if KEYOPT(5) = 0} \\ \dfrac{P_1 \, L \, C^A}{2} & \text{if KEYOPT(5) = 1} \end{cases}$$

$F_2 = F_8 = \dfrac{P_2 \, L \, C^A}{2}$

$F_3 = F_9 = \dfrac{P_3 \, L \, C^A}{2}$

$F_4 = F_{10} = 0.0$

$F_5 = -F_{11} = \dfrac{P_3 \, L^2 \, C^A}{12}$

$F_6 = -F_{12} = \dfrac{P_2 \, L^2 \, C^A}{12}$

P_1 = parallel pressure component in element coordinate system (force/unit length)

P_2, P_3 = transverse pressure components in element coordinate system (force/unit length)

$$C^A = \begin{cases} 1.0 & \text{if KEYOPT(5) = 0} \\ \text{positive sine of the angle between the axis of the element and the direction of the pressures, as defined by } P_1, P_2, \text{ and } P_3 & \text{if KEYOPT(5) = 1} \end{cases}$$

The transverse pressures are assumed to act on the centerline, and not on the inner or outer surfaces. The transverse pressures in the element coordinate system are computed by

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \end{Bmatrix} = [T] \begin{Bmatrix} P_X \\ P_Y \\ P_Z \end{Bmatrix} \qquad (14.16\text{–}9)$$

where:
- $[T]$ = conversion matrix defined in equation (14.4–6)
- P_X = transverse pressure acting in global Cartesian X direction) (input quantity PX)
- P_Y = transverse pressure acting in global Cartesian Y direction) (input quantity PY)
- P_Z = transverse pressure acting in global Cartesian Z direction) (input quantity PZ)

ϵ_x^{pr}, the unrestrained axial strain caused by internal and external pressure effects, is needed to compute the pressure part of the element load vector (see Figure 14.16–1).

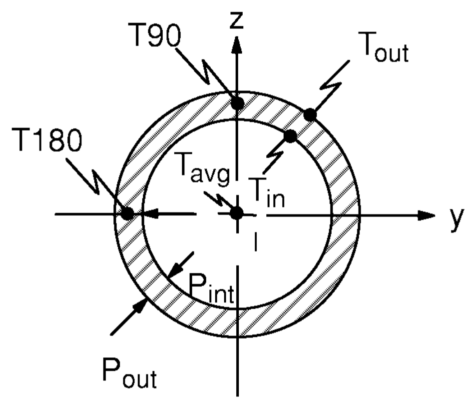


Figure 14.16–1 Thermal and Pressure Effects

ϵ_x^{pr} is computed using thick wall (Lame') effects:

$$\epsilon_x^{pr} = \frac{1}{E}(1 - 2\nu)\left(\frac{P_i D_i^2 - P_o D_o^2}{D_o^2 - D_i^2}\right) \qquad (14.16\text{–}10)$$

where:
- ν = Poisson's ratio (input as PRXY or NUXY on **MP** command)
- P_i = internal pressure (input on **SFE** command)
- P_o = external pressure (input on **SFE** command)

An element thermal load vector is computed also, based on thick wall effects.

14.16.8 Stress Calculation

The output stresses, computed at the outside surface and illustrated in Figure 14.16–2 and Figure 14.16–3, are calculated from the following definitions:

$$\sigma_{dir} = \frac{F_x + \frac{\pi}{4}\left(P_i\ D_i^{\ 2} - P_o\ D_o^2\right)}{a^w} \qquad (14.16\text{–}11)$$

$$\sigma_{bend} = C_\sigma \frac{M_b\ r_o}{I_r} \qquad (14.16\text{–}12)$$

$$\sigma_{tor} = \frac{M_x\ r_o}{J} \qquad (14.16\text{–}13)$$

$$\sigma_h = \frac{P_i D_i - P_o D_o}{2t_c} \qquad (14.16\text{–}14)$$

$$\sigma_{\ell f} = \frac{2\ F_s}{A^w} \qquad (14.16\text{–}15)$$

where:

σ_{dir} = direct stress (output quantity SDIR)

F_x = axial force

a_w = $\frac{\pi}{4}\left(d_o^{\ 2} - D_i^{\ 2}\right)$

d_o = $2\ r_o$

r_o = $\frac{D_o}{2} - t_c$

t_c = corrosion allowance (input quantity TKCORR on **RMORE** command)

σ_{bend} = bending stress (output quantity SBEND)

C_σ = stress intensification factor, defined in Table 14.16–1

M_b = bending moment = $\sqrt{M_y^2 + M_z^2}$

I_r = $\frac{\pi}{64}\left(d_o^{\ 4} - D_i^{\ 4}\right)$

σ_{tor} = torsional shear stress (output quantity ST)

M_x = torsional moment

J = $2I_r$

σ_h = hoop pressure stress (output quantity SH)

R_i = $\frac{D_i}{2}$

t_e = $t_w - t_c$

$\sigma_{\ell f}$ = lateral force shear stress (output quantity SSF)

F_s = shear force = $\sqrt{F_y^2 + F_z^2}$

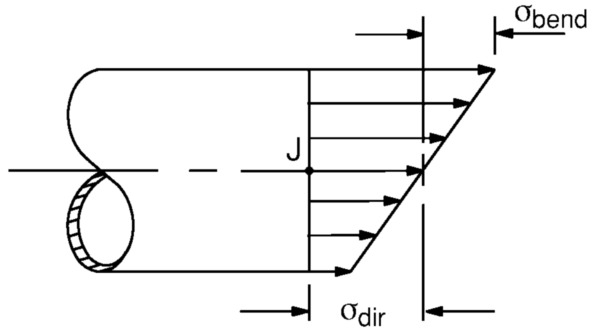


Figure 14.16–2 Elastic Pipe Output

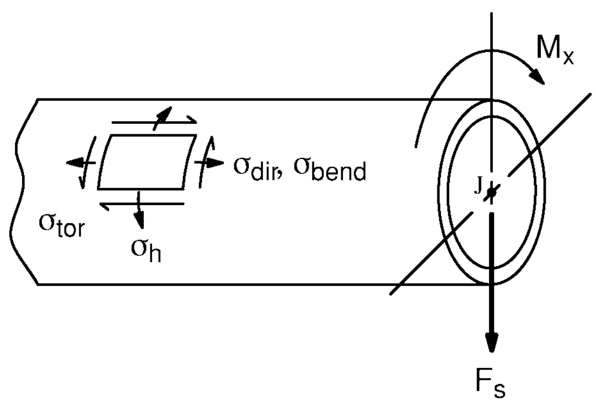


Figure 14.16–3 Elastic Pipe Output

Stress intensification factors are given in Table 14.16–1.

Table 14.16–1 Stress Intensification Factors

KEYOPT(2)	C_σ	
	At node I	At node J
0	$C_{\sigma,I}$	$C_{\sigma,J}$
1	$C_{\sigma,T}$	1.0
2	1.0	$C_{\sigma,T}$
3	$C_{\sigma,T}$	$C_{\sigma,T}$

Any entry in Table 14.16–1 either input as or computed to be less than 1.0 is set to 1.0. The entries are:

$C_{\sigma,I}$ = input quantity SIFI on **R** command

$C_{\sigma,J}$ = input quantity SIFJ on **R** command

$C_{\sigma,T}$ = "T" stress intensification factor (ASME Code(40))

$$= \frac{0.9}{\left(\frac{4\,t_w}{(D_i + d_o)}\right)^{2/3}}$$

σ_{th} (output quantity STH), which is in the postprocessing file, represents the stress due to the thermal gradient thru the thickness. If the temperatures are given as nodal temperatures, σ_{th} = 0.0. But, if the temperatures are input as element temperatures,

$$\sigma_{th} = -\frac{E\alpha\,(T_o - T_a)}{1 - \upsilon} \qquad (14.16\text{–}16)$$

where: T_o = temperature at outside surface

T_a = temperature midway thru wall

Equation (14.16–16) is derived as a special case of equations (2.1–12), (2.1–13) and (2.1–15) with y as the hoop coordinate (h) and z as the radial coordinate (r). Specifically, these equations

1. are specialized to an isotropic material
2. are premultiplied by [D] and –1
3. have all motions set to zero, hence $\varepsilon_x = \varepsilon_h = \varepsilon_r = \gamma_{xh} = \gamma_{hr} = \gamma_{xr} = 0.0$
4. have $\sigma_r = \tau_{hr} = \tau_{xr} = 0.0$ since r = R_o is a free surface.

This results in:

$$\begin{Bmatrix} \sigma_x^t \\ \sigma_h^t \\ \sigma_{xh}^t \end{Bmatrix} = \begin{bmatrix} -\frac{E}{1-\nu^2} & -\frac{\upsilon E}{1-\nu^2} & 0 \\ -\frac{\nu E}{1-\nu^2} & -\frac{E}{1-\nu^2} & 0 \\ 0 & 0 & -G \end{bmatrix} \begin{Bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ 0 \end{Bmatrix} \qquad (14.16\text{–}17)$$

or

$$\sigma_x^t = \sigma_h^t = -\frac{E\alpha\Delta T}{1 - \nu} = \sigma_{th} \qquad (14.16\text{–}18)$$

and

$$\sigma_{xh}^{t} = 0 \tag{14.16–19}$$

Finally, the axial and shear stresses are combined with:

$$\sigma_x = \sigma_{dir} + A\ \sigma_{bend} + \sigma_{th} \tag{14.16–20}$$

$$\sigma_{xh} = \sigma_{tor} + B\ \sigma_{\ell f} \tag{14.16–21}$$

where:
- A, B = sine and cosine functions at the appropriate angle
- σ_x = output quantity SAXL
- σ_{xh} = output quantity SXH

The maximum and minimum principal stresses, as well as the stress intensity and the equivalent stress, are based on the stresses at two extreme points on opposite sides of the bending axis, as shown in Figure 14.16–4. If shear stresses due to lateral forces $(\sigma_{\ell f})$ are greater than the bending stresses, the two points of maximum shearing stresses due to those forces are reported instead. The stresses are calculated from the typical Mohr's circle approach in Figure 14.16–5.

The equivalent stress for Point 1 is based on the three principal stresses which are designated by small circles in Figure 14.16–5. Note that one of the small circles is at the origin. This represents the radial stress on the outside of the pipe, which is equal to zero (unless $P_o \neq 0.0$). Similarly, the points marked with an X represent the principal stresses associated with Point 2, and a second equivalent stress is derived from them.

Next, the program selects the largest of the four maximum principal stresses (σ_1, output quantities S1MX), the smallest of the four minimum principal stresses (σ_3, output quantity S3MN), the largest of the four stress intensities (σ_I, output quantity SINTMX), and the largest of the four equivalent stresses (σ_e, output quantity SEQVMX). Finally, these are also compared (and replaced as necessary) to the values at the right positions around the circumference at each end. These four values are then printed out and put on the postprocessing file.

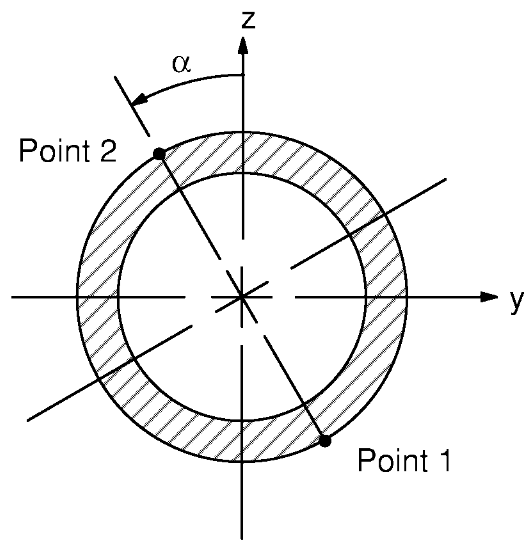


Figure 14.16–4 Stress Point Locations

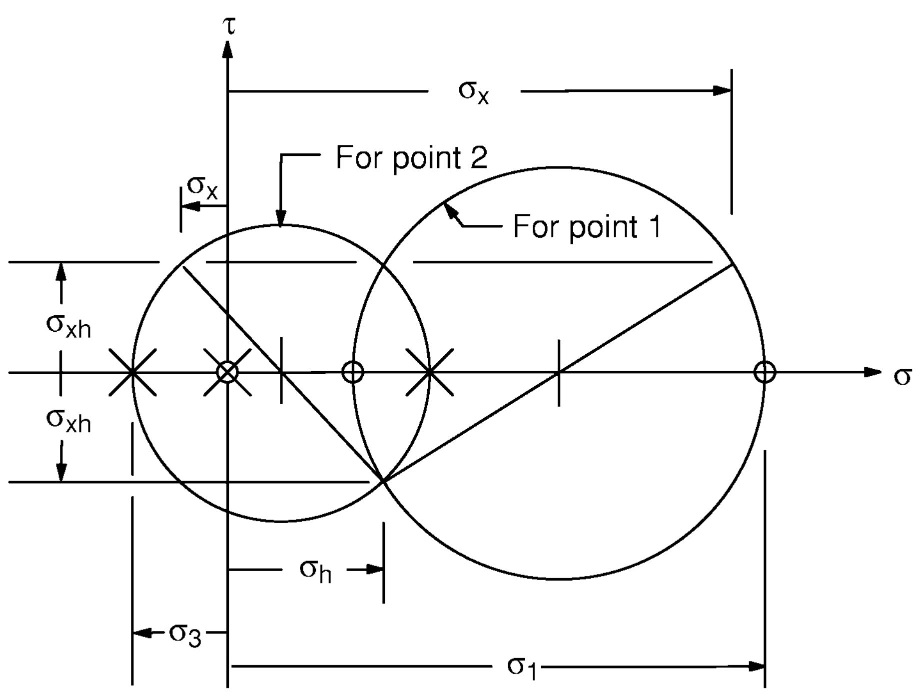


Figure 14.16–5 Mohr Circles

Three additional items are put on the postdata file for use with certain code checking. These are:

$$\sigma^c_{pr} = \frac{P_i D_o}{4t_w} \qquad (14.16\text{–}22)$$

$$\sigma_{MI} = \sqrt{M_{XI}^2 + M_{YI}^2 + M_{ZI}^2}\ \frac{D_o}{2I} \tag{14.16–23}$$

$$\sigma_{MJ} = \sqrt{M_{XJ}^2 + M_{YJ}^2 + M_{ZJ}^2}\ \frac{D_o}{2I} \tag{14.16–24}$$

where:
- σ_{pr}^c = output quantity SPR2
- σ_{MI} = output quantity SMI
- σ_{MJ} = output quantity SMJ
- M_{XI} = moment about the x axis at node I, etc.