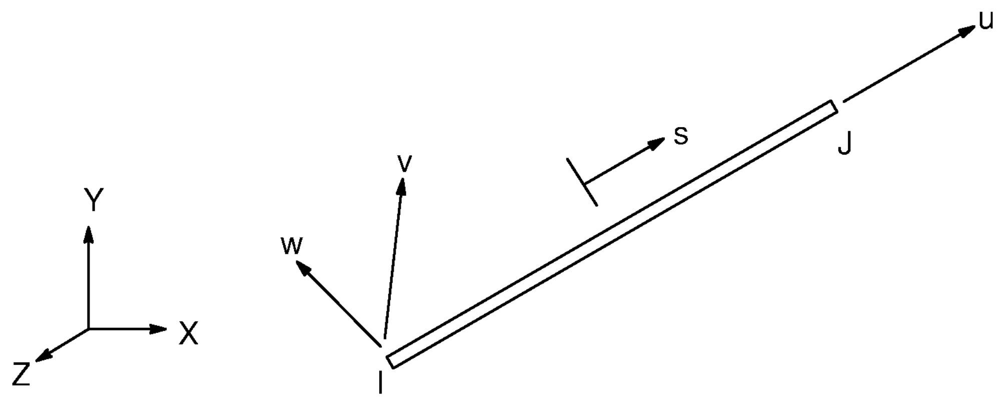
14.8 LINK8 — 3–D Spar (or Truss)



Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equation (12.2.1–1)	None
Mass Matrix	Equations (12.2.1–1), (12.2.1–2), and (12.2.1–3)	None
Stress Stiffening Matrix	Equations (12.2.1–2) and (12.2.1–3)	None
Thermal Load Vector	Equation (12.2.1–1)	None

Load Type	Distribution
Element Temperature	Linear along length
Nodal Temperature	Linear along length

Reference: Cook et al(117)

14.8.1 Assumptions and Restrictions

The element is not capable of carrying bending loads. The stress is assumed to be uniform over the entire element.

14.8.2 Element Matrices and Load Vector

All element matrices and load vectors described below are generated in the element coordinate system and are then converted to the global coordinate system. The element stiffness matrix is:

$$[K_\ell] = \frac{A\hat{E}}{L}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad (14.8\text{–}1)$$

where:

A = element cross–sectional area (input as AREA on **R** command)

$\hat{E}$ = { E, Young's modulus (input as EX on **MP** command) if linear.
E_T, tangent modulus (see Section 4.1) if plasticity is present and the tangent matrix is to be computed (see Section 4.1 and 4.4).

L = element length

The element mass matrix with **LUMPM**,OFF is:

$$[M_\ell] = \frac{\rho AL(1-\epsilon^{in})}{6}\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 & 0 & 1 \\ \hline 1 & 0 & 0 & 2 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2 \end{array}\right] \qquad (14.8\text{–}2)$$

where:

ϱ = density (input as DENS on **MP** command)

ε^{in} = initial strain (input as ISTRN on **R** command)

The element mass matrix with **LUMP**,ON is:

$$[M_\ell] = \frac{\rho AL(1-\epsilon^{in})}{2}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}\right] \quad (14.8\text{–}3)$$

The element stress stiffness matrix is:

$$[S_\ell] = \frac{F}{L}\left[\begin{array}{ccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \quad (14.8\text{–}4)$$

where: F =
- for the first iteration: $A\,E\,\varepsilon^{in}$
- for all subsequent iterations: the axial force in the element as computed in the previous stress pass of the element

The element load vector is:

$$\{F_\ell\} = \{F_\ell^a\} - \{F_\ell^{nr}\} \quad (14.8\text{–}5)$$

where: $\{F_\ell^a\}$ = the applied load vector

$\{F_\ell^{nr}\}$ = the Newton–Raphson restoring force, if applicable.

The applied load vector is:

$$\{F_\ell^a\} = AE\epsilon_n^T \ \lfloor -1\ 0\ 0\ 1\ 0\ 0 \rfloor^T \quad (14.8\text{–}6)$$

For a linear analysis or the first iteration of a nonlinear (Newton–Raphson) analysis ϵ_n^T is:

$$\epsilon_n^T = \epsilon_n^{th} - \epsilon^{in} \tag{14.8–7}$$

with

$$\epsilon_n^{th} = \alpha_n \left(T_n - T_{ref}\right)$$

where:
- α_n = coefficient of thermal expansion (input as ALPX on **MP** command) evaluated at T_n
- T_n = average temperature of the element in this iteration
- T_{ref} = reference temperature (input on **TREF** command)

For the subsequent iterations of a Newton–Raphson analysis:

$$\epsilon_n^T = \Delta\epsilon_n^{th} \tag{14.8–8}$$

with the thermal strain increment computed through:

$$\Delta\epsilon_n^{th} = \alpha_n \left(T_n - T_{ref}\right) - \alpha_{n-1} \left(T_{n-1} - T_{ref}\right)$$

where:
- α_n, α_{n-1} = coefficients of thermal expansion evaluated at T_n and T_{n-1}, respectively
- T_n, T_{n-1} = average temperature of the element for this iteration and the previous iteration

The Newton–Raphson restoring force vector is:

$$\left\{F_\ell^{nr}\right\} = AE\epsilon_{n-1}^{el} \quad \lfloor -1 \ 0 \ 0 \ 1 \ 0 \ 0 \ \rfloor^T \tag{14.8–9}$$

where:
- ϵ_{n-1}^{el} = the elastic strain for the previous iteration.

14.8.3 Force and Stress

For a linear analysis or the first iteration of a nonlinear (Newton–Raphson) analysis:

$$\epsilon_n^{el} = \epsilon_n - \epsilon_n^{th} + \epsilon^{in} \tag{14.8–10}$$

where:
- ϵ_n^{el} = elastic strain (output quantity EPELAXL)
- ϵ_n = total strain = $\frac{u}{L}$
- u = difference of nodal displacements in axial direction

ϵ_n^{th} = thermal strain (output quantity EPTHAXL)

For the subsequent iterations of a nonlinear (Newton–Raphson) analysis:

$$\epsilon_n^{el} = \epsilon_{n-1}^{el} + \Delta\epsilon - \Delta\epsilon^{th} - \Delta\epsilon^{pl} - \Delta\epsilon^{cr} - \Delta\epsilon^{sw} \tag{14.8–11}$$

where:
$\Delta\varepsilon$ = strain increment = $\frac{\Delta u}{L}$
Δu = difference of nodal displacements increment in axial direction
$\Delta\varepsilon^{th}$ = thermal strain increment
$\Delta\varepsilon^{pl}$ = plastic strain increment
$\Delta\varepsilon^{cr}$ = creep strain increment
$\Delta\varepsilon^{sw}$ = swelling strain increment

The stress is:

$$\sigma = E\,\epsilon^{a} \tag{14.8–12}$$

where:
σ = stress (output quantity SAXL)
ε^{a} = adjusted strain = $\epsilon_n^{el} + \Delta\varepsilon^{cr} + \Delta\varepsilon^{sw}$

Thus, the strain used to compute the stress has the creep and swelling effects as of the beginning of the substep, not the end. Finally,

$$F = A\sigma \tag{14.8–13}$$

where:
F = force (output quantity MFORX)