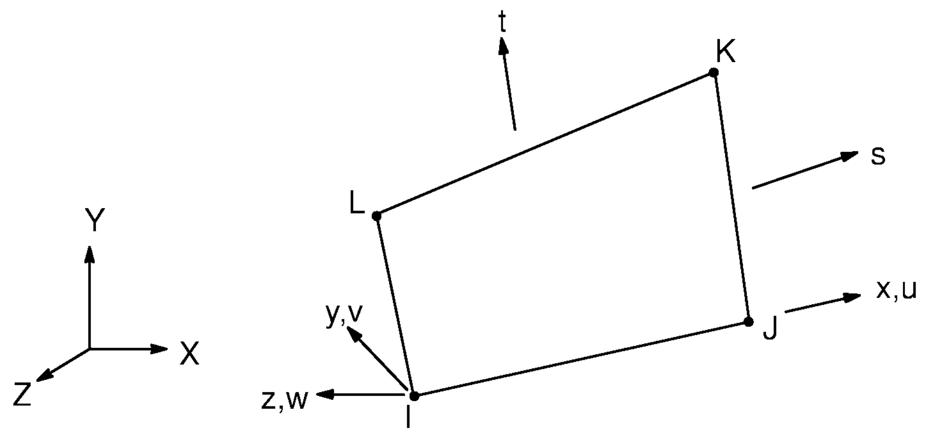
14.115 INTER115 — 3–D Magnetic Interface



Matrix or Vector	Geometry	Shape Functions	Integration Points
Coefficient Matrix	Quad	Equation (12.5.8–7), (12.5.8–8), (12.5.8–9), and (12.5.8–22)	2 x 2
	Triangle	Equation (12.5.1–7), (12.5.1–8), (12.5.1–9), and (12.5.1–22)	1
Load Vector	Same as coefficient matrix		Same as coefficient matrix

14.115.1 Element Matrix Derivation

Problem Description

A general 3–D electromagnetics problem is schematically shown in Figure 14.115–1. The analysis region of the problem may be divided into three parts. Ω_1 is the region of conduction, in which the conductivity, σ, is not zero so that eddy currents may be induced. Ω_1 may also be a ferromagnetic region so that the permeability μ is much larger than that of the free space, μ_o. However, no source currents exist in Ω_1. Both Ω_2 and Ω_3 are regions free of eddy currents. There may be source currents present in these regions. A distinction is made between Ω_2 and Ω_3 to ensure that the scalar potential region, Ω_3, is single–connected and to provide an option to place the source currents in either the vector potential or the scalar potential region. Γ_B and Γ_H represent boundaries on which fluxes are parallel and normal respectively.

In Ω_1, due to the non–zero conductivity and / or high permeability, the magnetic vector potential together with the electric scalar potential are employed to model the influence of eddy currents. In Ω_2, only the magnetic vector potential is used. In Ω_3, the total magnetic field is composed of a reduced field which is derived from the magnetic reduced scalar potential, ϕ, and the field, H_s, which is computed using the Biot–Savart law.

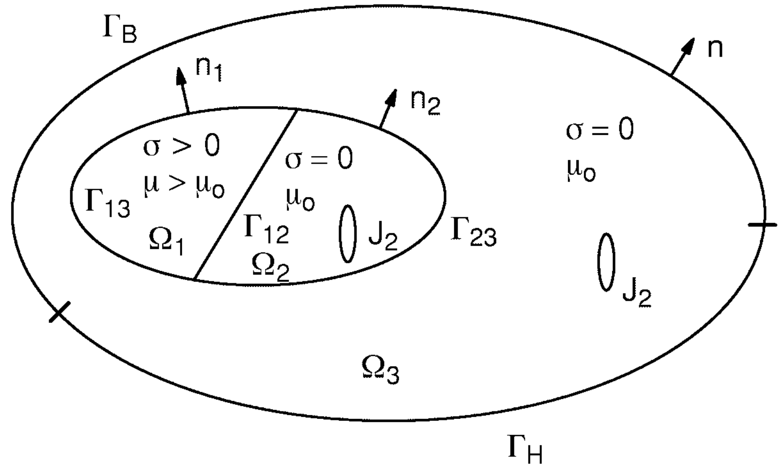


Figure 14.115–1 A General Electromagnetics Analysis Field and Its Component Regions

14.115.2 The A, V–A–ϕ Formulation

The equations relating the various field quantities are constituted by the following subset of Maxwell's equations with the displacement currents neglected.

$$\left.\begin{aligned} \nabla \times \{H\} - \{J_s\} - \{J_e\} &= \{0\} \\ \nabla \times \{E\} + \left\{\frac{\partial B}{\partial t}\right\} &= \{0\} \\ \nabla \cdot \{B\} &= 0 \end{aligned}\right\} \quad \text{in } \Omega_1 \qquad (14.115\text{–}1)$$

$$\left.\begin{aligned} \nabla \times \{H\} &= \{J_s\} \\ \nabla \cdot \{B\} &= 0 \end{aligned}\right\} \quad \text{in } \Omega_2 \cup \Omega_3 \qquad (14.115\text{–}2)$$

The constitutive relationships are:

$$\{B\} = [\mu]\{H\} \tag{14.115–3}$$

The boundary and interface conditions, respectively, are:

$$\{B\}^T \cdot \{n\} = 0 \quad \text{on } \Gamma_B \tag{14.115–4}$$

$$\{H\} \times \{n\} = \{0\} \quad \text{on } \Gamma_H \tag{14.115–5}$$

$$\left.\begin{array}{l}\{B_1\}^T \cdot \{n_1\} + \{B_2\} \cdot \{n_2\} = 0 \\ \{H_1\} \times \{n_1\} + \{H_2\} \times \{n_2\} = \{0\}\end{array}\right\} \text{on } \Gamma_{12}, \Gamma_{13}, \text{and } \Gamma_{23} \tag{14.115–6}$$

Variables are defined in Section 5.1.

By introducing the magnetic vector potential, {A} (AX, AY, AZ), both in Ω_1 and Ω_2; the electric scalar potential V (VOLT) in Ω_1; and the generalized scalar potential ϕ_g (MAG) in Ω_3, the field quantities can be written in terms of various potentials as:

$$\{B\} = \nabla \times \{A\} \quad \text{in } \Omega_1 \text{ and } \Omega_2 \tag{14.115–7}$$

$$\{E\} = -\left\{\frac{\partial A}{\partial t}\right\} - \nabla V \quad \text{in } \Omega_1 \tag{14.115–8}$$

$$\{H\} = \{H_s\} - \nabla \phi_g \quad \text{in } \Omega_3 \tag{14.115–9}$$

In order to make the solution of potential {A} unique, the Columb gauge condition is applied to define the divergence of {A} in addition to its curl.

Substituting equations (14.115–7) through (14.115–9) into the field equations and the boundary conditions equations (14.115–1) through (14.115–6) and using the Galerkin form of the method of weighted residual equations, the weak form of the differential equations in terms of the potentials {A}, V and ϕ_g can be obtained. Through some algebraic manipulations and by applying the boundary as well as interface conditions, respectively, the finite element equations may be written as:

$$\int_{\Omega_1+\Omega_2} \left(\left(\nabla \times [N_A]^T\right)^T [\nu] \left(\nabla \times \{A\}\right) + [\nu] \left(\nabla \cdot [N_A]^T\right)^T \left(\nabla \cdot \{A\}\right) + [\sigma] [N_A]^T \cdot \left\{\frac{\partial A}{\partial t}\right\} + [\sigma] [N_A]^T \cdot \nabla \frac{\partial v}{\partial t} \right) d\Omega - \int_{\Gamma_{13}+\Gamma_{23}} [N_A]^T \cdot \left(\nabla \phi_g \times \{n_3\}\right) d\Gamma$$
$$= - \int_{\Gamma_{13}+\Gamma_{23}} [N_A]^T \cdot \left(\{H_s\} \times \{n_2\}\right) d\Gamma + \int_{\Omega_2} [N_A]^T \cdot \{J_2\} \, d\Omega \qquad (14.115\text{–}10)$$

$$\int_{\Omega_1} \left([\sigma] \nabla \{N\} \cdot \left\{\frac{\partial A}{\partial t}\right\} + [\sigma] \nabla \{N\} \cdot \nabla \frac{\partial v}{\partial t} \right) d\Omega = 0 \qquad (14.115\text{–}11)$$

$$-\int_{\Omega_3} [\mu] \left(\nabla \{N\}\right)^T \cdot \nabla\phi_g \, d\Omega + \int_{\Gamma_{23}} \{N\} \{n_2\} \cdot \left(\nabla \times \{A\}\right) d\Gamma + \int_{\Gamma_{13}} \{N\} \{n_1\} \cdot \left(\nabla \times \{A\}\right) d\Gamma = - \int_{\Omega_2} \left(\nabla \{N\}\right)^T \cdot [\mu] \{H_s\} \, d\Omega \qquad (14.115\text{–}12)$$

where:
- $[N_A]$ = matrix of element shape functions for {A}
- {N} = vector of element shape function for both V and ϕ
- v = related to the potential V as:

$$V = \frac{\partial v}{\partial t} \qquad (14.115\text{–}13)$$

A number of interface terms arise in the above equations because of the coupling of vector potential and scalar potential formulations across different regions. These are the terms that involve integration over the surface shared by two adjoining subregions and are given as:

$$I_1 = -\int_{\Gamma_{13}+\Gamma_{23}} [N_A] \cdot \left(\nabla \phi_g \times \{n_3\}\right) d\Gamma \qquad (14.115\text{–}14)$$

$$I_2 = -\int_{\Gamma_{13}+\Gamma_{23}} \{N\} \{n_3\} \cdot \left(\nabla \times \{A\}\right) d\Gamma \qquad (14.115\text{–}15)$$

$$I_3 = -\int_{\Gamma_{13}+\Gamma_{23}} [N_A] \cdot (\{H_s\} \times \{n_3\})\, d\Gamma \qquad (14.115\text{–}16)$$

where: Γ_{ij} = surface at the interface of subregions Ω_i and Ω_j, respectively.

The term, I_3, contributes to the load vector while the terms, I_1 and I_2, contribute to the coefficient matrix. The asymmetric contributions of I_1 and I_2 to the coefficient matrix may be made symmetric following the procedure by Emson and Simkin(176). After some algebraic manipulations including applying the Stokes' theorem, we get

$$I_2 = I_{21} + I_{22} \qquad (14.115\text{–}17)$$

$$I_{21} = -\int_{\Gamma_{13}+\Gamma_{23}} (\nabla \{N\} \times \{n_3\}) \cdot \{A\}\, d\Gamma \qquad (14.115\text{–}18)$$

$$I_{22} = -\oint_{\Gamma_{13}+\Gamma_{23}} \{N\} \{A\} \cdot d\bar{\ell} \qquad (14.115\text{–}19)$$

It is observed from equation (14.115–17) that the integrals represented by I_1 and I_2 are symmetric if the condition $I_{22} = 0$ is satisfied. The integral given by I_{22} is evaluated along a closed path lying on the interface. If the interface lies completely inside the region of the problem, the integrals over the internal edges will cancel each other; if the integral path is on a plane of symmetry, the tangential component of {A} will be zero, so the integral will be vanish; and if the integral path is on the part of the boundary where the scalar potential is prescribed, the terms containing N will be omitted and the symmetry of the matrix will be ensured. Therefore, the condition that ensures symmetry can usually be satisfied. Even if, as in some special cases, the condition can not be directly satisfied, the region may be remeshed to make the interface of the vector and scalar potential regions lie completely inside the problem domain. Thus, the symmetry condition can be assumed to hold without any loss of generality.

Replacing the vector and scalar potentials by the shape functions and nodal degrees of freedom as described by equations (14.115–20) through (14.115–23),

$$\{A\} = [N_A]^T \{A_e\} \qquad (14.115\text{–}20)$$

$$\left\{\frac{\partial A}{\partial t}\right\} = [N_A]^T \{\dot{A}_e\} \qquad (14.115\text{–}21)$$

$$\phi_g = \{N\}^T \{\phi_e\} \qquad (14.115\text{–}22)$$

$$V = \frac{\partial v}{\partial t} = \{N\}^T\{V_e\} \tag{14.115–23}$$

the above manipulations finally result in the following set of finite element equations:

$$\int_{\Omega_1} \left[\left(\nabla \times [N_A]^T\right)^T [\nu] (\nabla \times [N_A]^T) + [\nu] \nabla \cdot [N_A]^T \nabla \cdot [N_A]^T\right] d\Omega \{A_e\}$$

$$+ \int_{\Omega_1} [\sigma] [N_A]^T \cdot [N_A] d\Omega \{\dot{A}_e\} + \int_{\Omega_1} [\sigma] [N_A]^T \cdot \nabla \{N\} d\Omega \{V_e\} \tag{14.115–24}$$

$$- \int_{\Gamma_{13}} [N_A]^T \cdot \left(\nabla \{N\} \times \{n_3\}\right) d\Gamma\{\phi_e\} = -\int_{\Gamma_{13}} [N_A]^T \cdot [N_A] \times \{n_3\} d\Gamma \{H_s\}$$

$$\int_{\Omega_1} [\sigma] \nabla \{N\}^T \cdot [N_A] d\Omega \{\dot{A}_e\} + \int_{\Omega_1} [\sigma] \nabla \{N\}^T \cdot \nabla \{N\} d\Omega \{V_e\} = 0 \tag{14.115–25}$$

$$\int_{\Omega_2} \left[[\nu] (\nabla \times [N_A])^T \cdot (\nabla \times [N_A]) + [\nu] \nabla \cdot [N_A] \nabla \cdot [N_A]\right] d\Omega \{A_e\}$$

$$-\int_{\Gamma_{23}} [N_A]^T \cdot (\nabla \{N\} \times \{n_3\}) d\Gamma \{\phi_e\} \tag{14.115–26}$$

$$= -\int_{\Gamma_{23}} [N_A]^T \cdot [N_A] \times \{n_3\} d\Gamma\{H_s\} + \int_{\Omega_2} [N_A]^T[N_A] d\Omega \{J_2\}$$

$$-\int_{\Omega_3} [\mu] \nabla \{N\}^T \cdot \nabla \{N\} d\Omega \{\phi_e\} - \int_{\Gamma_{13}+\Gamma_{23}} (\nabla \{N\} \times \{n_3\})^T \cdot [N_A] d\Gamma \{A_e\}$$

$$= -\int_{\Omega_2} [\mu] \nabla \{N\}^T[N_A] d\Omega \{H_s\} \tag{14.115–27}$$

Equations (14.115–24) through (14.115–27) represent a symmetric system of equations for the entire problem.

The interface elements couple the vector potential and scalar potential regions, and therefore have AX,AY, AZ and MAG degrees of freedom at each node. The coefficient matrix and the load vector terms in equations (14.115–24) through (14.115–27) are computed in the magnetic vector potential elements (SOLID97), the scalar potential elements SOLID96, SOLID98 with KEYOPT(1) = 10, or SOLID5 with KEYOPT(1) = 10) and the interface elements (INTER115). The only terms in these equations that are computed in the interface elements are given by:

Coefficient Matrix:

$$[K] = -\int_{\Gamma_{13}+\Gamma_{23}} [N_A]^T \cdot (\nabla \{N\} \times \{n_3\})\, d\Gamma - \int_{\Gamma_{13}+\Gamma_{23}} (\nabla \{N\} \times \{n_3\})^T \cdot [N_A]\, d\Gamma \qquad (14.115\text{–}28)$$

Load Vector:

$$\{F\} = -\int_{\Gamma_{13}+\Gamma_{23}} [N_A]^T \cdot ([N_A] \times \{n_3\})\, d\Gamma \qquad (14.115\text{–}29)$$