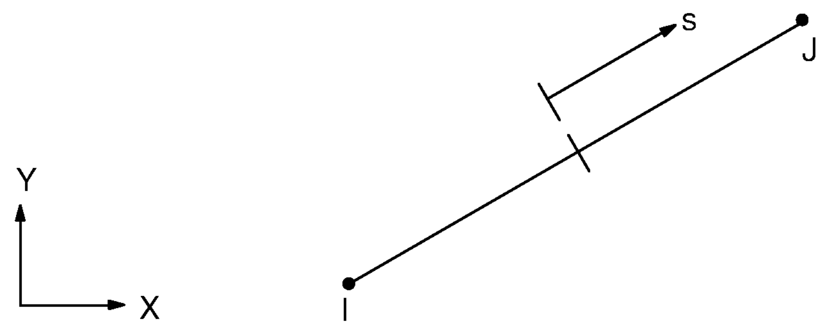
14.9 INFIN9 — 2–D Infinite Boundary



Matrix or Vector	Shape Functions	Integration Points
Magnetic Potential Coefficient Matrix or Thermal Conductivity Matrix	$A = C_1 + C_2x$	None

References: Kagawa, Yamabuchi and Kitagami(122)

14.9.1 Introduction

This boundary element (BE) models the exterior infinite domain of the far–field magnetic and thermal problems. This element is to be used in combination with elements having a magnetic potential (AZ) or temperature (TEMP) as the DOF.

14.9.2 Theory

The formulation of this element is based on a first order infinite boundary element (IBE) that is compatible with first order quadrilateral or triangular shaped finite elements, or higher order elements with dropped midside nodes. For unbounded field problems., the model domain is set up to consist of an interior finite element domain, Ω_F, and a series of exterior BE subdomains, Ω_B, as shown in Figure 14.9–1. Each subdomain, Ω_B, is treated as an ordinary BE domain consisting of four segments: the boundary element I–J, infinite elements J–K and I–L, and element K–L; element K–L is assumed to be located at infinity.

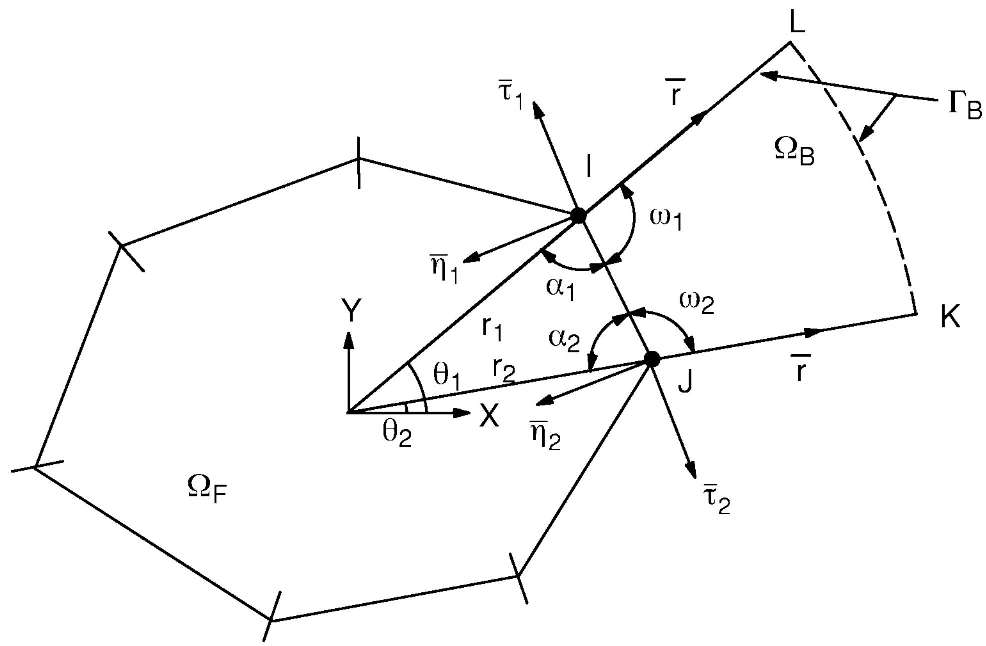


Figure 14.9–1 Definition of BE Subdomain and the Characteristics of the IBE

The approach used here is to write BE equations for Ω_B, and then convert them into equivalent load vectors for the nodes I and J. The procedure consists of four separate steps that are summarized below (see reference (122) for details).

First, a set of boundary integral equations is written for Ω_B. To achieve this, linear shape functions are used for the BE I–J:

$$N_1(s) = \frac{1}{2}\,(1 - s) \tag{14.9–1}$$

$$N_2(s) = \frac{1}{2}\,(1 + s) \tag{14.9–2}$$

Over the infinite elements J–K and I–L the potential (or temperature) ϕ and its derivative q (flux) are respectively assumed to be:

$$\phi(r) = \phi_i \left(\frac{r_i}{r}\right), \quad i = I, J \tag{14.9–3}$$

$$q(r) = q_i \left(\frac{r_i}{r}\right)^2, \quad i = I, J \tag{14.9–4}$$

The boundary integral equations are the same as presented in equation (14.47–5) except that the Green's function in this case would be:

$$G(x,\xi) = \frac{1}{2\pi k} \ln\left(\frac{\sqrt{k}}{r}\right) \qquad (14.9–5)$$

where:

x = field point in boundary element

ξ = source point

k = magnetic reluctivity (inverse of free space permeability input on **EMUNIT** command) for AZ DOF (KEYOPT(1)=0)

or

thermal conductivity (input as KXX on **MP** command) for TEMP DOF (KEYOPT(1)=1)

Note that all the integrations in the present case are performed in closed form.

Second, in the absence of a source or sink in Ω_B, the flux q(r) is integrated over the boundary Γ_B of Ω_B and set to zero.

$$\int_{\Gamma_B} q \, d\Gamma = 0 \qquad (14.9–6)$$

Third, a geometric constraint condition that exists between the potential ϕ and its derivatives $\frac{\partial\phi}{\partial n} = q_n$ and $\frac{\partial\phi}{\partial\tau} = q_\tau$ at the nodes I and J is written as:

$$q_{n_i} = q_{\tau_i} \cos\alpha_i + \phi_i \frac{\sin\alpha_i}{r_i} \quad i = I, J \qquad (14.9–7)$$

Fourth, the energy flow quantity from Ω_B is written as:

$$w = \int_{\Gamma_B} q \, \phi \, d\Gamma \qquad (14.9–8)$$

This energy flow is equated to that due to an equivalent nodal {F} defined below.

The four steps mentioned above are combined together to yield, after eliminating q_n and q_τ,

$$[K]\{\phi\} = \{F\} \qquad (14.9–9)$$

where: [K] = 2 x 2 equivalent unsymmetric element coefficient matrix
{ϕ} = 2 x 1 nodal DOFs, AZ or TEMP
{F} = 2 x 1 equivalent nodal force vector

For linear problems, the INFIN9 element forms the coefficient matrix [K] only. The load vector {F} is not formed. The coefficient matrix multiplied by the nodal DOF's represents the nodal load vector which brings the effects of the semi–infinite domain Ω_B onto nodes I and J.