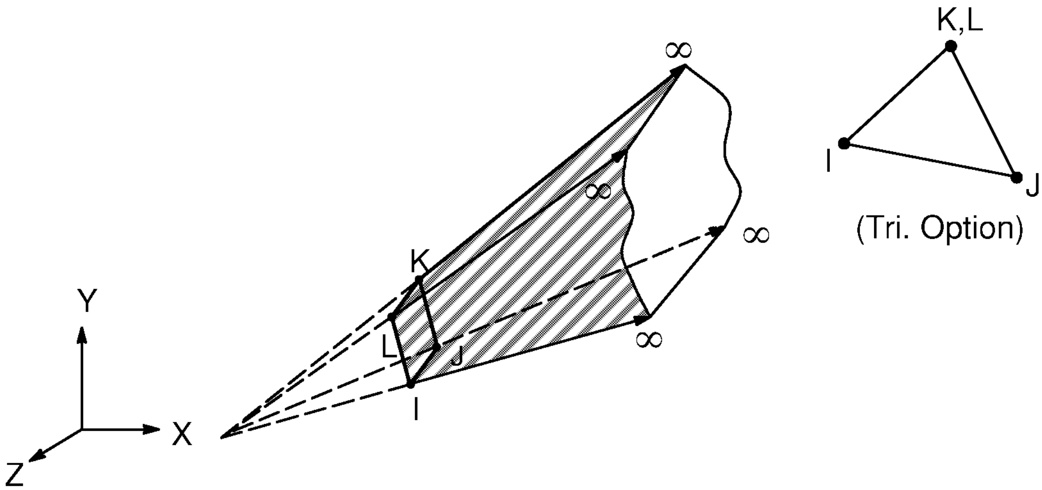
14.47 INFIN47 — 3–D Infinite Boundary



Matrix or Vector	Shape Functions	Integration Points
Magnetic Potential Coefficient Matrix or Thermal Conductivity Matrix	$\phi = N_I\phi_I + N_J\phi_J + N_K\phi_K,$ $N_I = \frac{1}{2A_o}\left[(x_J y_K - x_K y_J) - (y_K - y_J)x + (x_K - x_J)y\right]$ $N_J = \frac{1}{2A_o}\left[(x_K y_I - x_I y_K) - (y_I - y_K)x + (x_I - x_K)y\right]$ $N_K = \frac{1}{2A_o}\left[(x_I y_J - x_J y_I) - (y_J - y_I)x + (x_J - x_I)y\right]$ A_o = area of triangle IJK	None on the boundary element IJK itself, however, 16–point 1–D Gaussian quadrature is applied for some of the integration on each of the edges IJ, JK, and KI of the infinite elements IJML, JKNM, and KILN (see Figure 14.47–1)

Reference: Kaljevic′, et al(130)

14.47.1 Introduction

This boundary element (BE) models the exterior infinite domain of the far–field magnetic and thermal problems. This element is to be used in combination with 3–D scalar potential solid elements, and can have magnetic scalar potential (MAG), or temperature (TEMP) as the DOF.

14.47.2 Theory

The formulation of this element is based on a first order triangular infinite boundary element (IBE), but the element can be used as a 4–noded quadrilateral as well. For unbounded field problems, the model domain is set up to consist of an interior volumetric finite element domain, Ω_F, and a series of exterior volumetric BE subdomains, Ω_B, as shown in Figure 14.47–1. Each subdomain, Ω_B, is treated as an ordinary BE domain consisting of five segments: the boundary element IJK, infinite elements IJML, JKNM and KILN, and element LMN; element LMN is assumed to be located at infinity (Figure 14.47–1).

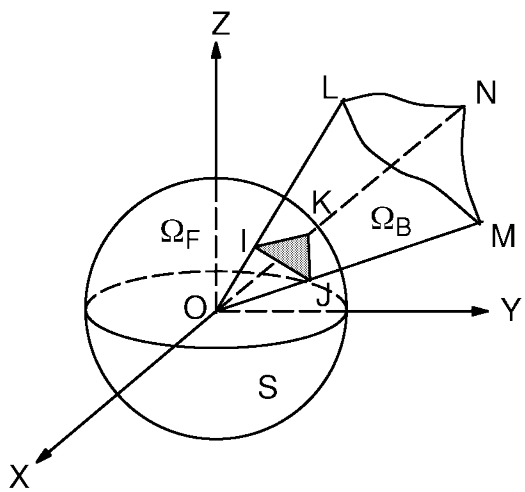


Figure 14.47–1 A Semi–infinite Boundary Element Zone and the Corresponding Boundary Element IJK

The approach used here is to write BE equations for Ω_B, and then convert them into equivalent load vectors for the nodes I, J and K. The procedure consists of four steps that are summarized below (see (130) for details).

First, a set of boundary integral equations is written for Ω_B. To achieve this, the potential (or temperature) and its normal derivatives (fluxes) are interpolated on the triangle IJK (Figure 14.47–1) by linear shape functions:

$$\phi(x, y) = N_I \ \phi_I + N_J \ \phi_J \ N_K \ \phi_K \tag{14.47–1}$$

$$q_n(x, y) = N_I \, q_{nI} + N_J \, q_{nJ} + N_K \, q_{nK} \tag{14.47–2}$$

where:
- ϕ = potential (or temperature)
- q_n = $\frac{\partial \phi}{\partial n}$ = normal derivative (or flux)
- N_I, N_J, N_K = linear shape functions defined earlier
- ϕ_I, ϕ_J, ϕ_K = nodal potentials (or temperatures)
- q_{nI}, q_{nJ}, q_{nK} = nodal normal derivatives (or fluxes)
- n = normal to the surface IJK

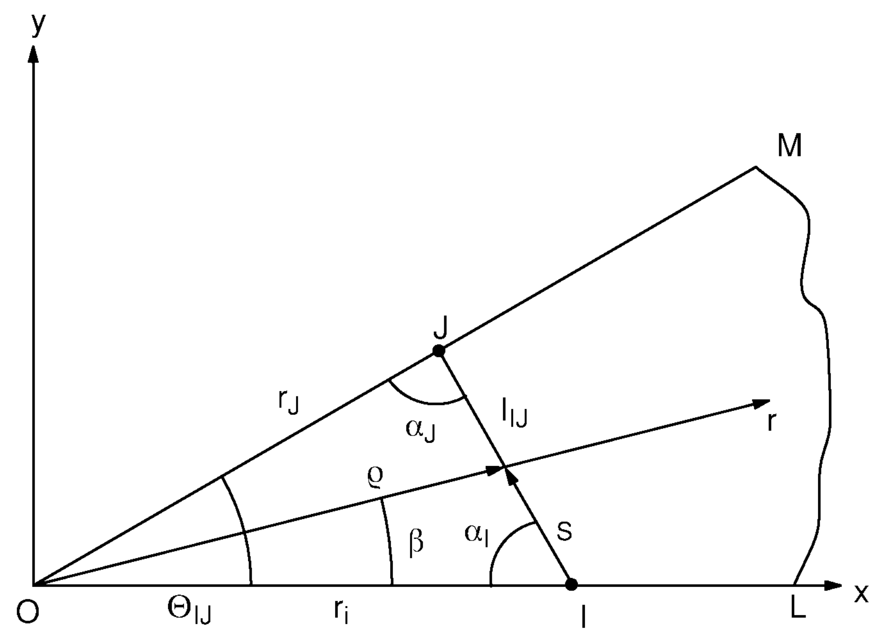


Figure 14.47–2 Infinite Element IJML (see Figure 14.47–1) and the Local Coordinate System

Over an infinite element, such as IJML (Figure 14.47–2), the dependent variables, i.e., potentials (or temperatures) and their normal derivatives (fluxes) are respectively assumed to be (Figure 14.47–2):

$$\phi(r, \beta) = \left\{ \left(1 - \frac{s}{L_{IJ}}\right)\phi_I + \left(\frac{s}{L_{IJ}}\right)\phi_J \right\} \left(\frac{\rho}{r}\right)^2 \tag{14.47–3}$$

$$q_\tau(r, \beta) = \left\{ \left(1 - \frac{s}{L_{IJ}}\right)q_{\tau J} + \left(\frac{s}{L_{IJ}}\right)q_{\tau J} \right\} \left(\frac{\rho}{r}\right)^3 \tag{14.47–4}$$

where: q_τ = $\frac{\partial \phi}{\partial \tau}$ = normal derivative (or flux) to infinite elements, e.g., IJML (Figure 14.47–2)

$q_{\tau I}, q_{\tau J}$ = nodal (nodes I and J) normal derivatives for infinite element IJML

s = a variable length from node I towards node J

L_{IJ} = length of edge IJ

ϱ = radial distance from the origin of the local coordinate system O to the edge IJ

r = radial distance from the edge IJ towards infinity

β = variable angle from x–axis for local polar coordinate system

τ = normal to infinite elements, viz., IJML

The boundary integral equations for Ω_B are now written as:

$$c(\xi)\phi(\xi) = \int_{\Gamma_B} \left[G(x,\xi)\; q(x) - F(x,\xi)\; \phi(x)\right] d\Gamma(x) \tag{14.47–5}$$

where: $c(\xi)$ = jump term in boundary element method

$G(x,\xi)$ = Green's function or fundamental solution for Laplace's equation

= $\frac{1}{4\pi k r}$

$F(x,\xi)$ = $\frac{\partial}{\partial n}[G(x,\xi)]$

(x,ξ) = field and source points, respectively

r = distance between field and source points

k = { magnetic reluctivity (inverse of free space permeability input on **EMUNIT** command) for AZ DOF (KEYOPT(1)=0)

or

isotropic thermal conductivity (input as KXX on **MP** command) for TEMP DOF (KEYOPT(1)=1)

The integrations in equations (14.47–5) are performed in closed form on the boundary element IJK. The integrations on the infinite elements IJML, JKNM and KILN in the 'r' direction (Figure 14.47–2) are also performed in closed form. However, a 16–point Gaussian quadrature rule is used for the integrations on each of the edges IJ, JK and KI on the infinite elements.

Second, in the absence of a source or sink in Ω_B, the flux q(r) is integrated over the boundary Γ_B of Ω_B and set to zero:

$$\int_{\Gamma_B} q \; dr = 0 \tag{14.47–6}$$

Third, geometric constraint conditions that exist between the potential ϕ (or temperature) and its derivatives $\frac{\partial \phi}{\partial n} = q_n$ and $\frac{\partial \phi}{\partial \tau} = q_\tau$ at the nodes I, J and K are written. These conditions would express the fact that the normal derivative q_n at the node I, say, can be decomposed into components along the normals to the two infinite elements IJML and KILN meeting at I and along OI.

Fourth, the energy flow quantity from Ω_B is written as:

$$w = \int_{\Gamma_B} q \; \phi \; dr \tag{14.47–7}$$

This energy flow is equated to that due to an equivalent nodal force vector {F} defined below.

The four steps mentioned above are combined together to yield, after eliminating q_n and q_τ,

$$[K] \; \{\phi\} \; \equiv \; \{F\}_{eqv} \tag{14.47–8}$$

where:

$[K]$ = 3 x 3 equivalent unsymmetric element coefficient matrix

$\{\phi\}$ = 3 x 1 nodal degrees of freedom, MAG or TEMP

$\{F\}_{eqv}$ = 3 x 1 equivalent nodal force vector

The coefficient matrix [K] multiplied by the nodal DOF's $\{\phi\}$ represents the equivalent nodal load vector which brings the effects of the semi–infinite domain Ω_B onto nodes I, J and K.

As mentioned in the beginning, the INFIN47 can be used with magnetic scalar potential elements to solve 3–D magnetic scalar potential problems (MAG degree of freedom). Magnetic scalar potential elements incorporate three different scalar potential formulations (see Section 5.1) selected with the **MAGOPT** command:

No.	Formulation	MAGOPT
1	Reduced Scalar Potential	0
2	Difference Scalar Potential	2 and 3
3	Generalized Scalar Potential	1, 2 and 3

Reduced Scalar Potential

If there is no “iron” in the problem domain, the reduced scalar potential formulation can be used both in the FE and the BE regimes. In this case, the potential is continuous across FE–BE interface. If there is “iron” in the FE domain, the reduced potential formulation is likely to produce “cancellation errors”.

Difference Scalar Potential

If there is “iron” and current in the FE region and the problem domain is singly–connected, we can use the difference potential formulation in order to avoid cancellation error. The formulation consists of two–step solution procedures:

1. **MAGOPT**, 2 Solution

   Here the first step consists of computing a magnetic field $\{H_o\}$ under the assumption that the magnetic permeability of iron is infinity, thereby neglecting any saturation. The reduced scalar potential ϕ is used in FE region and the total scalar potential ψ is used in BE region. In this case, the potential will be discontinuous across the FE–BE interface. The continuity condition of the magnetic field at the interface can be written as:

   $$-\nabla\psi\cdot\{\tau\} = -\nabla\phi\cdot\{\tau\} + \{H_s\}^T\{\tau\} \quad (14.47\text{–}9)$$

   where:

   $\{\tau\}$ = tangent vector at the interface along element edge

   $\{H_s\}$ = magnetic field due to current sources

   Integrating the above equation along the interface, we obtain

   $$\psi_p = \phi_p - \int_{p_o}^{p} \{H_s\}^T\{\tau\}\,dt \quad (14.47\text{–}10)$$

   If we take $\psi = \phi$ at a convenient point p_o on the interface, then the above equation defines the potential jump at any point p on the interface. Now, the total potential ψ can be eliminated from the problem using this equation, leading to the computation of the additional load vector,

   $$\{f_g\} = [K]\{g\} \quad (14.47\text{–}11)$$

   where:

   $g_i = \int_{p_o}^{p_i} \{H_s\}^T\{\tau\}\,dt$

   [K] = coefficient matrix defined with equation (14.47–8)

2. **MAGOPT**, 3 Solution

   In this step the total field, $\{H\} = \{H_o\} - \nabla\psi$, is computed where {H} is the actual field and $\{H_o\}$ is the field computed in step 1 above. Note that the same relation given in equation (5.1–38) uses ϕ_g in place of ψ. The total potential ψ is used in both FE and BE regimes. As a result, no potential discontinuity exists at the interface, but an additional load vector due to the field $\{H_o\}$ must be computed. Since the magnetic flux continuity condition at the interface of air and iron is:

$$\mu_I \frac{\partial \psi_I}{\partial n} - \mu_o \frac{\partial \psi_A}{\partial n} = - \mu_o \{H_o\}^T \{n\} \tag{14.47–12}$$

   where:
   μ_o = magnetic permeability of free space (air)
   μ_I = magnetic permeability of iron

   The additional load vector may be computed as

$$\{f_f\} = - \int_s \mu_o \{N\}\{H_o\}^T \{n\} \, ds \tag{14.47–13}$$

   where:
   {N} = weighting functions

Generalized Scalar Potential

If there is iron and current in the FE domain and the domain is multiply–connected, the generalized potential formulation can be used. It consists of three different steps.

1. **MAGOPT**, 1 Solution. This step computes magnetic field in the iron only. The boundary elements are not at all used for this step.
2. **MAGOPT**, 2 Solution. This step is exactly the same as the step 1 of the difference potential formulation.
3. **MAGOPT**, 3 Solution. This step is exactly the same as the step 2 of the difference potential formulation.