14.110 INFIN110 — 2–D Infinite Solid
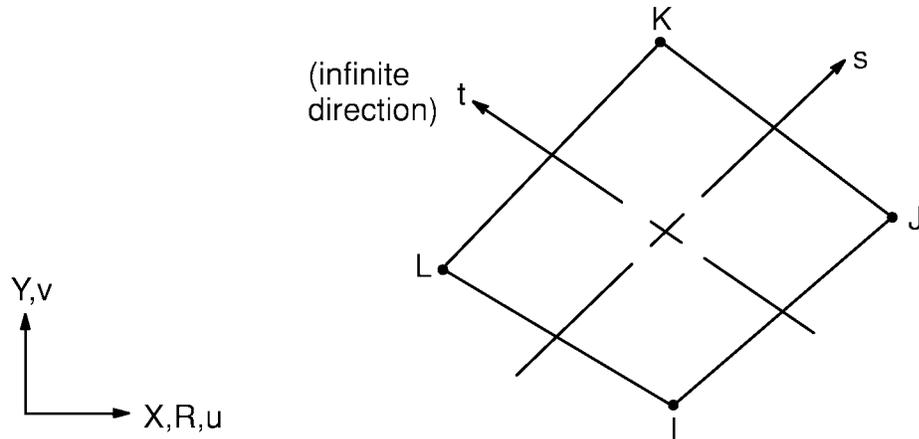
Matrix or Vector	Mapping and Shape Functions	Integration Points
Magnetic Potential Coefficient Matrix	Equations (12.6.8–9), (12.6.8–33), and (12.6.8–34)	2 x 2
Thermal Conductivity Matrix	Equations (12.6.8–20), (12.6.8–33), and (12.6.8–34)	2 x 2
Specific Heat Matrix	Equations (12.6.8–20), (12.6.8–33), and (12.6.8–34)	2 x 2
Electrical Potential Coefficient Matrix	Equations (12.6.8–21), (12.6.8–33), and (12.6.8–34)	2 x 2

References: Zienkiewicz et al.(169), Damjanic' and Owen(170), Marques and
Owen(171), Li et al.(172)

14.110.1 Mapping Functions

The theory for the infinite mapping functions is briefly summarized here. Consider the
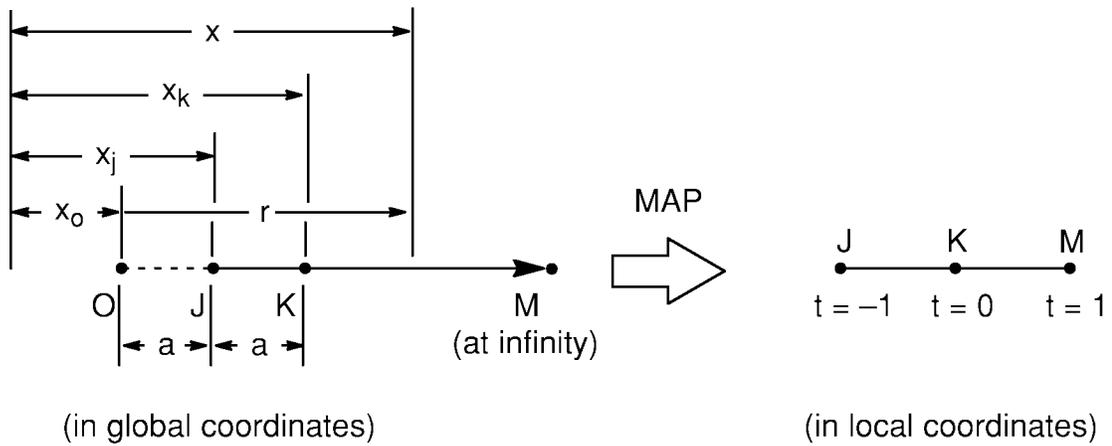one–dimensional situation shown in Figure 14.110–1

Figure 14.110–1 Global to Local Mapping of a One–Dimensional Infinite Element

The one–dimensional element may be thought of as one edge of the infinite element of Figure 14.110–2. It extends from node J, through node K to the point M at infinity and is mapped onto the parent element defined by the local coordinate system in the range −1 ≤ t ≤ 1.
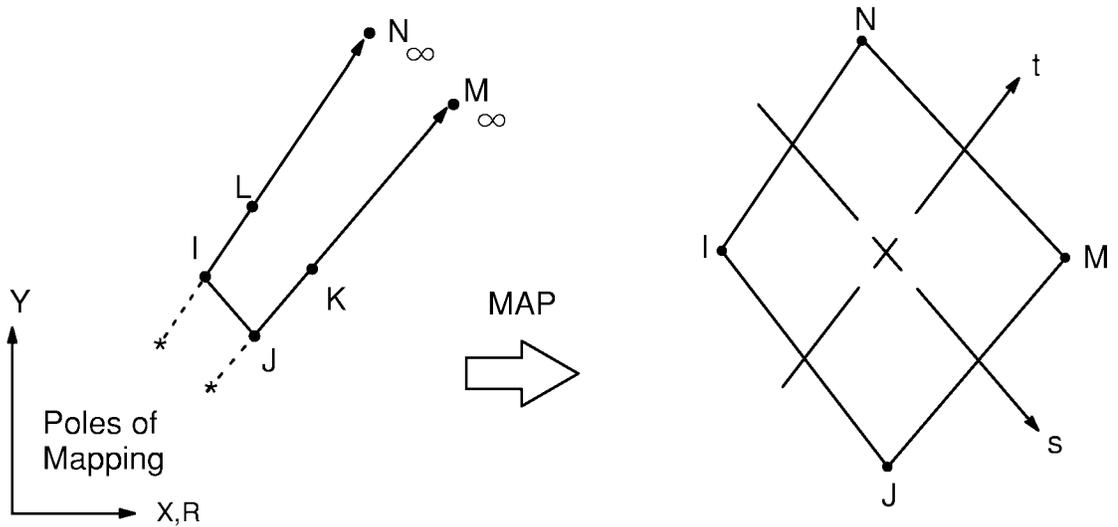

Figure 14.110–2 Mapping of 2–D Solid Infinite Element

The position of the "pole", x_o, is arbitrary, and once chosen, the location of node K is defined by

$$x_K = 2x_J - x_o \tag{14.110-1}$$

The interpolation from local to global positions is performed as

$$x(t) = M_J(t) \ x_J + M_K(t) \ x_K \tag{14.110-2}$$

where:
$$M_J(t) = -2t / (1 - t)$$
$$M_K(t) = 1 - M_J(t)$$

Examining the above mapping, it can be seen that t = −1, 0, 1 correspond respectively to the global positions x = x_J, x_K, ∞, respectively.

The basic field variable, A (A_z for KEYOPT(1) = 0, VOLT for KEYOPT(1) = 1 or TEMP for KEYOPT(1) = 2) can be interpolated using standard shape functions, which when written in polynomial form becomes

$$A(t) = b_0 + b_1 t + b_2 t^2 + b_3 t^3 + - - - \tag{14.110-3}$$

Solving equation (14.110–2) for t yields

$$t = 1 - \frac{2a}{r} \tag{14.110-4}$$

where:
r = distance from the pole, O, to a general point within the element

a = $x_K - x_J$ as shown in Figure 14.110–2

Substituting (14.110–4) into (14.110–3) gives

$$A(t) = c_0 + \frac{c_1}{r} + \frac{c_2}{r^2} + \frac{c_3}{r^3} + - - - \tag{14.110-5}$$

Where c_0 = 0 is implied since the variable A is assumed to vanish at infinity.

The expression (14.110–5) is truncated at the quadratic (r^2) term in the present implementation. Expression (14.110–5) also shows the role of the pole position, O.

In 2–D (Figure 14.110–2) mapping is achieved by the shape function products. The mapping functions and the Lagrangian isoparametric shape functions for 2–D and axisymmetric 4 node quadrilaterals are given in Section 12.6.8. The shape functions for the nodes M and N are not needed as the field variable, A, is assumed to vanish at infinity.

14.110.2   Matrices

The matrices can be written as:

$$[K_e] = \int_{vol} [B]^T [D] \ [B] \ d(vol) \tag{14.110-6}$$

$$[C_e] = \int_{vol} C_c \ \{N\} \ \{N\}^T \ d(vol) \tag{14.110-7}$$

where: $\{N\}$ =   shape functions given in Section 12.6.8

              $[B]$ =   defined later

1. AZ DOF (KEYOPT(1) = 0)

$[K_e]$ =   magnetic potential coefficient matrix

$$[D] \quad = \quad \frac{1}{\mu_o} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

μ_o =   magnetic permeability of free space (input on **EMUNIT** command)

The infinite elements can be used in magnetodynamic analysis even though these elements do not compute mass matrices. This is because air has negligible conductivity.

2. VOLT DOF (KEYOPT(1) = 1)

$[K_e]$ =   electrical potential coefficient matrix

$$[D] \quad = \quad \begin{bmatrix} \epsilon_x & 0 \\ 0 & \epsilon_y \end{bmatrix}$$

ϵ_x, ϵ_y =   electrical permitivity (input as PERX and PERY on **MP** command)

3. TEMP DOF (KEYOPT(1) = 2)

$[K_e]$ =   thermal conductivity matrix

$[C_e]$ =   specific heat matrix

$$[D] \quad = \quad \begin{bmatrix} k_x & 0 \\ 0 & k_y \end{bmatrix}$$

k_x, k_y =   thermal conductivities in the x and y direction (input as KXX and KYY on **MP** command)

C_c = $\rho \ C_p$

ρ =   density of the fluid (input as DENS on **MP** command)

C_p =   specific heat of the fluid (input as C on **MP** command)

Although it is assumed that the nodal DOFs are zero at infinity, it is possible to solve thermal problems in which the nodal temperatures tend to some constant value, T_o, rather than zero. In that case, the temperature differential, θ (= $T - T_o$), may be thought to be posed as the nodal DOF. The actual temperature can then be easily found from T = $\theta + T_o$. For transient analysis, θ must be zero at infinity t > 0, where t is time. Neumann boundary condition is automatically satisfied at infinity. Note that with these infinite elements, meaningful steady–state thermal analysis can be performed only when the problem is driven by heat sources or sinks (**BF** or **BFE** command with *Lab* = HGEN).

The $\{B_i\}$ vectors of the [B] matrix in equation (14.110–6) contain the derivatives of N_i with respect to the global coordinates which are evaluated according to

$$\{B_i\} = \begin{Bmatrix} \dfrac{\partial N_i}{\partial x} \\ \dfrac{\partial N_i}{\partial y} \end{Bmatrix} = [J]^{-1} \begin{Bmatrix} \dfrac{\partial N_i}{\partial s} \\ \dfrac{\partial N_i}{\partial t} \end{Bmatrix} \tag{14.110–8}$$

where: [J]   =   Jacobian matrix which defines the geometric mapping

[J] is given by

$$[J] = \sum_{i=1}^{4} \begin{bmatrix} \dfrac{\partial M_i}{\partial s} x_i & \dfrac{\partial M_i}{\partial s} y_i \\ \dfrac{\partial M_i}{\partial t} x_i & \dfrac{\partial M_i}{\partial t} y_i \end{bmatrix} \tag{14.110–9}$$

The mapping functions [M] in terms of s and t are given in Section 12.6.8. The domain differential d(vol) must also be written in terms of the local coordinates, so that

$$d(vol) = dx\ dy = |J|\ ds\ dt \tag{14.110–10}$$

Subject to the evaluation of $\{B_i\}$ and d(vol), which involves the mapping functions, the element matrices $[K_e]$ and $[C_e]$ may now be computed in the standard manner using Gaussian quadrature.