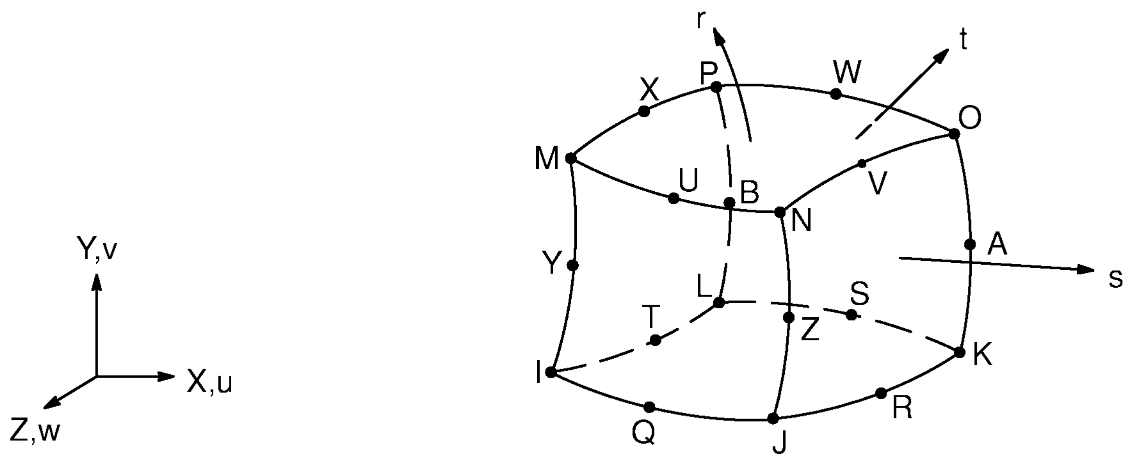
14.120 HF120 — High–Frequency Brick Solid



Matrix or Vector	Geometry	Geometric Shape Functions	Solution Shape Functions	Integration Points
Stiffness, Mass and Damping Matrices	Brick	Equations (12.8.20–1), (12.8.20–2), and (12.8.20–3)	Polynomial variable in order from 1 to 2	Variable
	Wedge	Equations (12.8.14–1), (12.8.14–2), and (12.8.14–3)	Polynomial variable in order from 1 to 2	Variable

Matrix or Vector	Geometry	Geometric Shape Functions	Solution Shape Functions	Integration Points
Surface PORT, INF, IMPD, SHLD Load Vectors	Quad	Equations (12.5.10–1) and (12.5.10–2)	Polynomial variable in order from 1 to 2	Variable
	Triangle	Equations (12.5.2–1) and (12.5.2–2)	Polynomial variable in order from 1 to 2	Variable

Load Type	Distribution
Surface Loads	Bilinear across each face

14.120.1 Other Applicable Sections

Section 5.6 describes the derivation of element matrices and load vectors as well as result evaluations.

14.120.2 Solution Shape Functions – H(curl) Conforming Element

HF120 uses a set of vector solution functions, which belong to the finite element functional space, H(curl), introduced by Nedelec(158). These vector functions have, among others, a very useful property, i.e., they possess tangential continuity on the boundary between two adjacent elements. This property fits naturally the need of HF120 to solve the electric field E based on the Maxwell's equations, since E is only tangentially continuous across material interfaces.

Let W_i,, i=1,,,N_V be such vector shape functions defined in the brick element. The electric field E is approximated by:

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N_v} E_i \vec{W}_i (\vec{r}) \qquad (14.120\text{–}1)$$

where:

$\vec{r}$ = position vector within the element.

N_V = number of vector shape functions

E_i = covariant components of E

In the following, three aspects in equation (14.120–1) are explained, i.e., how to define the W_i functions, how to choose the number of functions N_v, and what are the physical meanings of the associated expansion coefficients E_i. Recall that coefficients E_i are represented by the AX DOF in HF120.

To proceed, a few geometric definitions associated with an oblique coordinate system are necessary. Refer to the brick element shown at the beginning of this subsection. The geometry of the element is determined by the following mapping:

$$\vec{r} = \sum_{j=1}^{20} N_i\,(s,\ t,\ r)\ \vec{r}_i \tag{14.120–2}$$

where:

N_i = standard isoparametric shape functions

$\vec{r}_i$ = global coordinates for the 20 nodes.

Based on the mapping, a set of unitary basis vectors can be defined (Stratton(209)):

$$\vec{a}_1 = \frac{\partial \vec{r}}{\partial s} \qquad \vec{a}_2 = \frac{\partial \vec{r}}{\partial t} \qquad \vec{a}_3 = \frac{\partial \vec{r}}{\partial r} \tag{14.120–3}$$

These are simply tangent vectors in the local oblique coordinate system (s, t, r). Alternatively, a set of reciprocal unitary basis vectors can also be defined:

$$\begin{cases} \vec{a}^{\,1} = \dfrac{\vec{a}_2 \times \vec{a}_3}{J} & \vec{a}^{\,2} = \dfrac{\vec{a}_3 \times \vec{a}_1}{J} \\[2ex] \vec{a}^{\,3} = \dfrac{\vec{a}_1 \times \vec{a}_2}{J} & J = \vec{a}_1 \cdot \vec{a}_2 \times \vec{a}_3 \end{cases} \tag{14.120–4}$$

A vector F may be represented using either set of basis vectors:

$$\vec{F} = \sum_{i=1}^{3} f^i\,\vec{a}_i = \sum_{j=1}^{3} f_j\,\vec{a}^{\,j} \tag{14.120–5}$$

where

f_j = covariant components

f^i = contravariant components.

Given the covariant components of a vector F, its curl is found to be

$$\nabla \times \vec{F} = \frac{1}{J}\begin{vmatrix} \vec{a}_1 & \vec{a}_2 & \vec{a}_3 \\ \dfrac{\partial}{\partial s} & \dfrac{\partial}{\partial t} & \dfrac{\partial}{\partial r} \\ f_1 & f_2 & f_3 \end{vmatrix} \tag{14.120–6}$$

Having introduced the above geometric concepts, appropriate vector shape functions for the brick element are defined next. For the first order element (KEYOPT(1) = 1), there is one function associated with each edge:

$$\vec{w}_i = \begin{cases} \phi_i \, \vec{a}^{\,1}, & i = Q,\ S,\ U,\ W \\ \phi_i \, \vec{a}^{\,2}, & i = R,\ T,\ V,\ X \\ \phi_i \, \vec{a}^{\,3}, & i = Y,\ Z,\ A,\ B \end{cases} \qquad (14.120\text{–}7)$$

where: ϕ_i = scalar functions.

Therefore, N_v=12.

Now consider the second order brick (KEYOPT(1)=2). There are two functions defined for each edge. For example for node Q:

$$\vec{w}_i^{\,(1)} = \phi_i^{(1)} \, \vec{a}^{\,1}, \qquad \vec{w}_i^{\,(2)} = \phi_i^{(2)} \, \vec{a}^{\,1}. \qquad (14.120\text{–}8)$$

In addition, there are two functions defined associated with each face of the brick. For example, for the face MNOP (r = 1):

$$\vec{w}_f^{\,(1)} = \phi_f^{(1)} \, \vec{a}^{\,1}, \qquad \vec{w}_f^{\,(2)} = \phi_f^{(2)} \, \vec{a}^{\,2}. \qquad (14.120\text{–}9)$$

The total number of functions are N_v = 36.

Since each vector functions W_i has only one covariant component, it becomes clear that each expansion coefficients E_i in (1), i.e., the AX DOF, represents a covariant component of the electric field E at a proper location, aside from a scale factor that may apply. The curl of E can be readily computed by using equation (14.120–6).

Similarly, we can define vector shape functions for the wedge shape by combining functions from the brick and tetrahedral shapes. See Section 14.119.2 for tetrahedral functions.