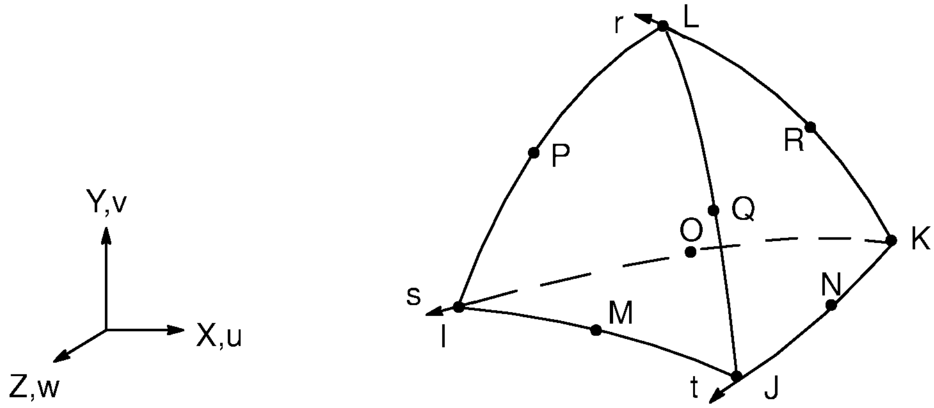
14.119 HF119 — 3–D High–Frequency Tetrahedral Solid



Matrix or Vector	Geometric Shape Functions	Solution Shape Functions	Integration Points
Stiffness, Mass and Damping Matrices	Equations (12.8.2–1), (12.8.2–2) and (12.8.2–3)	Polynomial variable in order of 1	Variable
Surface PORT, INF, IMPD, SHLD Load Vectors	Equations (12.5.2–1) and (12.5.2–2)	Polynomial variable in order of 1	Variable

Load Type	Distribution
Surface Loads	Linear across each face

14.119.1 Other Applicable Sections

Section 5.6 describes the derivation of element matrices and load vectors as well as results evaluations.

14.119.2 Solution Shape Functions – H (curl) Conforming Elements

HF119, along with HF120, uses a set of vector solution functions, which belong to the finite element functional space, H(curl), introduced by Nedelec(158). These vector functions have, among others, a very useful property, i.e., they possess tangential continuity on the boundary between two adjacent elements. This property fits naturally the need of HF119 to solve the electric field E based on the Maxwell's equations, since E is only tangentially continuous across material interfaces.

Similar to HF120 as discussed in section 14.120.2, the electric field E is approximated by:

$$\vec{E}(\vec{r}) = \sum_{i=1}^{N_v} E_i \vec{W}_i (\vec{r}) \qquad (14.119\text{–}1)$$

where:

W_i = vector shape functions defined in the tetrahedral element

E_i = covariant components of E at proper locations (AX DOFs)

N_v = number of vector functions

Refer to the tetrahedral element shown at the beginning of this subsection. The geometry of the element is represented by the following mapping:

$$\vec{r} = \sum_{j=1}^{10} N_j (L_1, L_2, L_3, L_4) \vec{r}_j \qquad (14.119\text{–}2)$$

where:

N_j = nodal shape functions

L_j = volume coordinates

r_j = nodal coordinates

Consider the local oblique coordinate system (s, t, r) based on node K. A set of unitary vectors can be defined as:

$$\vec{a}_1 = \frac{\partial \vec{r}}{\partial L_1} - \frac{\partial \vec{r}}{\partial L_3} \quad \vec{a}_2 = \frac{\partial \vec{r}}{\partial L_2} - \frac{\partial \vec{r}}{\partial L_3} \quad \vec{a}_3 = \frac{\partial \vec{r}}{\partial L_4} - \frac{\partial \vec{r}}{\partial L_3} \qquad (14.119\text{–}3)$$

These defines subsequently the gradients of the four volume coordinates:

$$\begin{cases} \nabla L_1 = \dfrac{\vec{a}_2 \times \vec{a}_3}{J_t} & \nabla L_2 = \dfrac{\vec{a}_3 \times \vec{a}_1}{J_t} \\ \nabla L_4 = \dfrac{\vec{a}_1 \times \vec{a}_2}{J_t} & \nabla L_3 = -\nabla L_1 - \nabla L_2 - \nabla L_4 \\ J_t = \vec{a}_1 \cdot \vec{a}_2 \times \vec{a}_3 & \end{cases} \tag{14.119–4}$$

The vector shape functions for the first order tetrahedral element can be conveniently defined as

$$\vec{W}_{ij} = L_i \nabla L_j - L_j \nabla L_i \qquad i,\ j = I,\ J,\ K,\ L \qquad i \neq j \tag{14.119–5}$$

The first order element is often referred to as the Whitney element(Whitney (208)).