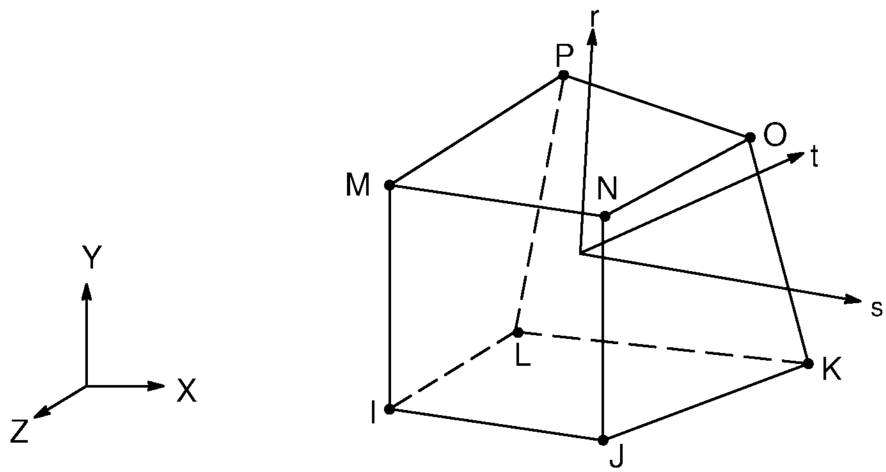
14.142 FLUID142 — 3–D Fluid



Matrix or Vector	Geometry	Shape Functions	Integration Points
Advection–Diffusion Matrix for Momentum Equations (X, Y and Z)	Brick, Pyramid, and Wedge	Equations (12.8.18–10), (12.8.18–11) and (12.8.18–12)	1 (default) or 2 x 2x 2 (adjustable with the **FLDA**,QUAD,MOMD command)
	Tet	Equations (12.8.18–10), (12.8.18–11) and (12.8.18–12)	1
Advection–Diffusion Matrix for Pressure	Brick, Pyramid, and Wedge	Equation (12.8.18–19)	Same as for equation momentum, but adjustable with the **FLDA**,QUAD,PRSD command
	Tet	Equation (12.8.18–19)	1

Matrix or Vector	Geometry	Shape Functions	Integration Points
Advection–Diffusion Matrix for Energy (Temperature)	Brick, Pyramid, and Wedge	Equation (12.8.18–20)	Same as for momentum, equations but adjustable with the **FLDA**,QUAD,THRD command
	Tet	Equation (12.8.18–20)	1
Advection–Diffusion Matrices for Turbulent Kinetic Energy and Dissipation Rate	Brick, Pyramid, and Wedge	Equations (12.8.18–23) and (12.8.18–24)	Same as for momentum, equations but adjustable with the **FLDA**,QUAD,TRBD command
	Tet	Equations (12.8.18–23) and (12.8.18–24)	1
Momentum Equation Source Vector	Brick, Pyramid, and Wedge	Equations (12.8.18–10), (12.8.18–11) and (12.8.18–12)	1 (default) or 2 x 2x 2 (adjustable with the **FLDA**,QUAD,MOMS command)
	Tet	Equations (12.8.18–10), (12.8.18–11) and (12.8.18–12)	1
Pressure Equation Source Vector	Brick, Pyramid, and Wedge	Equation (12.8.18–19)	Same as for equation momentum, but adjustable with the **FLDA**,QUAD,PRSS command
	Tet	Equation (12.8.18–19)	1
Heat Generation Vector	Brick, Pyramid, and Wedge	Equation (12.8.18–20)	Same as for momentum, equations but adjustable with the **FLDA**,QUAD,THRS command
	Tet	Equation (12.8.18–20)	1

Matrix or Vector	Geometry	Shape Functions	Integration Points
Turbulent Kinetic Energy and Dissipation Rate Source Term Vectors	Brick, Pyramid, and Wedge	Equations (12.8.18–23) and (12.8.18–24)	Same as for momentum, equations but adjustable with the **FLDA**,QUAD,TRBS command
	Tet	Equations (12.8.18–23) and (12.8.18–24)	1
Distributed Resistance Source Term Vector	Same as momentum equation source vector		Same as momentum equation source vector
Convection Surface Matrix and Load Vector and Heat Flux Load Vector	Brick, Pyramid, and Wedge	One–fourth of the element surface area times the heat flow rate is applied at each face node	None
	Tet	One–third of the element surface area times the heat flow rate is applied at each face node	

14.142.1 Other Applicable Sections

Chapter 7 describes the derivation of the applicable matrices, vectors, and output quantities. Chapter 6 describes the derivation of the heat transfer logic, including the film coefficient treatment.

14.142.2 Distributed Resistance Main Diagonal Modification

Suppose the matrix equation representation for the momentum equation in the X direction written without distributed resistance may be represented by the expression:

$$A_x^m \, V_x = b_x^m \qquad (14.142\text{–}1)$$

The source terms for the distributed resistances are summed:

$$D^{Rx} = \left[\rho \ K_x \ |V| + \frac{f_x \ \rho \ |V|}{D_{hx}} + C_x \ \mu \right] \tag{14.142–2}$$

where:

D^{Rx} = distributed resistance in the x direction
K_x = loss coefficient in the X direction
ϱ = density
f_x = friction factor for the X direction
μ = viscosity
C_x = permeability in the X direction
$|V|$ = velocity magnitude
D_{hx} = hydraulic diameter in the X direction

Consider the ith node algebraic equation. The main diagonal of the A matrix and the source terms are modified as follows:

$$A_{ii}^{mx} = A_{ii}^{mx} + D_i^{Rx} \tag{14.142–3}$$

$$b_i^{mx} = b_i^{mx} + 2 \ D_i^{Rx} \ V_x \tag{14.142–4}$$

14.142.3 Turbulent Kinetic Energy Source Term Linearization

The source terms are modified for the turbulent kinetic energy k and the turbulent kinetic energy dissipation rate ε to prevent negative values of kinetic energy.

The source terms for the kinetic energy combine as follows:

$$S_k = \mu_t \frac{\partial V_i}{\partial X_j} \left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i} \right) - \rho\epsilon \tag{14.142–5}$$

where the velocity spatial derivatives are written in indicial notation and μ_t is the turbulent viscosity:

$$\mu_t = C_\mu \rho \frac{k^2}{\epsilon} \tag{14.142–6}$$

where:

ϱ = density
C_μ = constant

The source term may thus be rewritten:

$$S_k = \mu_t \frac{\partial V_i}{\partial X_j} \left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i} \right) - C_\mu \rho^2 \frac{k^2}{\mu_t} \qquad (14.142\text{–}7)$$

A truncated Taylor series expansion of the kinetic energy term around the previous (old) value is expressed:

$$S_k = S_{k_{old}} + \left. \frac{\partial S_k}{\partial k} \right|_{k_{old}} (k - k_{old}) \qquad (14.142\text{–}8)$$

The partial derivative of the source term with respect to the kinetic energy is:

$$\frac{\partial S_k}{\partial k} = -2\ C_\mu \rho^2 \frac{k}{\mu_t} \qquad (14.142\text{–}9)$$

The source term is thus expresed

$$S_k = \mu_t \frac{\partial V_i}{\partial X_j} \left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i} \right) + C_\mu \rho^2 \frac{k_{old}^2}{\mu_t} - 2\ C_\mu \rho^2 \frac{k_{old}}{\mu_t} k \qquad (14.142\text{–}10)$$

The first two terms are the source term, and the final term is moved to the coefficient matrix. Denote by A^k the coefficient matrix of the turbulent kinetic energy equation before the linearization. The main diagonal of the ith row of the equation becomes:

$$A_{ii}^k = A_{ii}^k + 2\ C_\mu \rho^2 \frac{k_{old}}{\mu_t} \qquad (14.142\text{–}11)$$

and the source term is:

$$S_k = \mu_t \frac{\partial V_i}{\partial X_j} \left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i} \right) + C_\mu \rho^2 \frac{k_{old}^2}{\mu_t} \qquad (14.142\text{–}12)$$

14.142.4 Turbulent Kinetic Energy Dissipation Rate Source Term Linearization

The source term for the dissipation rate is handled in a similar fashion.

$$S_\epsilon = C_1 \mu_t \frac{\epsilon}{k} \frac{\partial V_i}{\partial X_j} \left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i} \right) - C_2 \rho \frac{\epsilon^2}{k} \qquad (14.142\text{–}13)$$

Replace ε using the expression for the turbulent viscosity to yield

$$S_\epsilon = C_1 C_\mu \rho k \frac{\partial V_i}{\partial X_j}\left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i}\right) - C_2 \rho \frac{\epsilon^2}{k} \qquad (14.142\text{–}14)$$

A truncated Taylor series expansion of the dissipation source term around the previous (old) value is expressed

$$S_\epsilon = S_{\epsilon\,old} + \left.\frac{\partial S_\epsilon}{\partial \epsilon}\right|_{\epsilon_{old}} (\epsilon - \epsilon_{old}) \qquad (14.142\text{–}15)$$

The partial derivative of the dissipation rate source term with respect to ε is:

$$\frac{\partial S_\epsilon}{\partial \epsilon} = -2\ C_2 \rho \frac{\epsilon}{k} \qquad (14.142\text{–}16)$$

The dissipation source term is thus expressed

$$S_\epsilon = C_1 C_\mu \rho k \frac{\partial V_i}{\partial X_j}\left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i}\right) + C_2 \rho \frac{\epsilon_{old}^2}{k} - 2\ C_2 \rho \frac{\epsilon_{old}}{k}\,\epsilon \qquad (14.142\text{–}17)$$

The first two terms are the source term, and the final term is moved to the coefficient matrix. Denote by A^ε the coefficient matrix of the turbulent kinetic energy dissipation rate equation before the linearization. The main diagonal of the ith row of the equation becomes:

$$A_{ii}^\epsilon = A_{ii}^\epsilon + 2\ C_2 \rho \frac{\epsilon_{old}}{k} \qquad (14.142\text{–}18)$$

and the source term is:

$$S_\epsilon = C_1\ C_\mu\ \rho\ k \frac{\partial V_i}{\partial X_j}\left(\frac{\partial V_i}{\partial X_j} + \frac{\partial V_j}{\partial X_i}\right) + C_\mu\ \rho \frac{\epsilon_{old}^2}{k} \qquad (14.142\text{–}19)$$