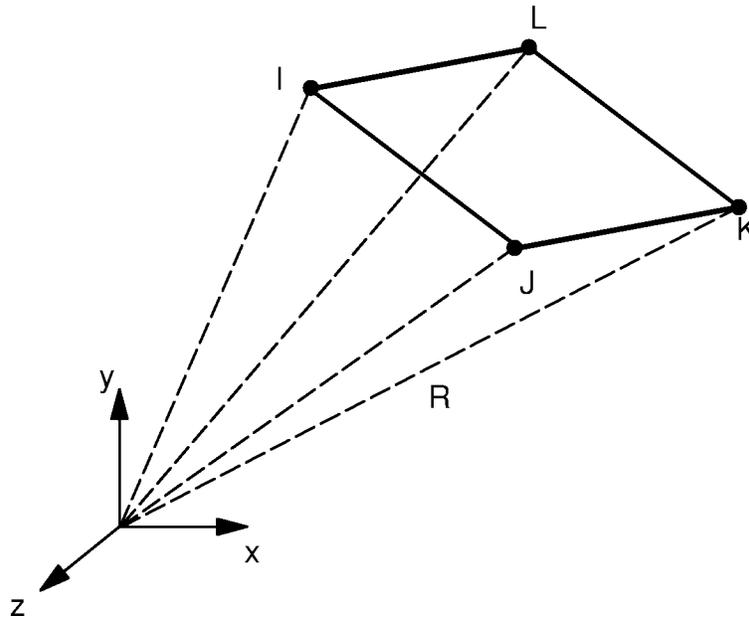


# 14.130 FLUID130 — 3-D Infinite Acoustic



Matrix or Vector	Shape Functions	Integration Points
Fluid Stiffness Matrix	Equation (12.6.5–19)	2 x 2
Fluid Damping Matrix	Same as Fluid Stiffness Matrix	2 x 2

## 14.130–1 Mathematical Formulation and F.E. Discretization

The exterior structural acoustics problem typically involves a structure submerged in an infinite, so-called acoustic fluid. The latter characterization implies that the fluid is homogeneous, linear, compressible and inviscid. When considering small barotropic disturbances of the pressure and density about an equilibrium state, it can be shown that the pressure field  $P$  [PRES] within the fluid is described by the scalar wave equation as:

$$\nabla^2 p = \frac{1}{c^2} \ddot{P} \quad \text{in } \Omega^+ \quad (14.130-1)$$

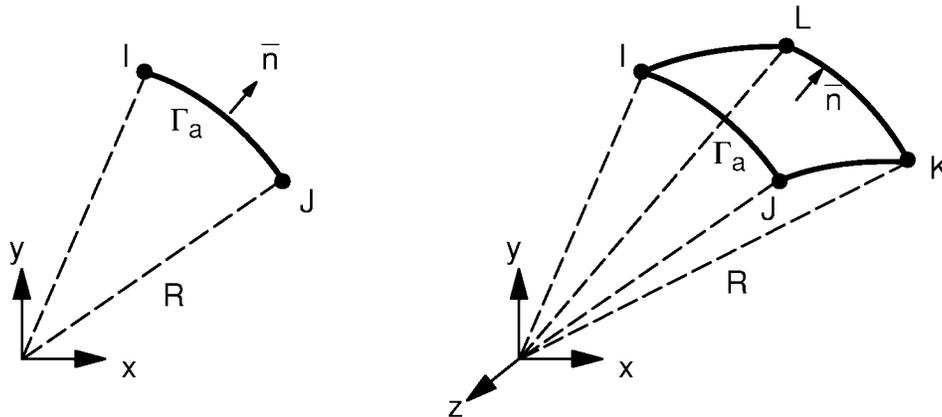
where:  $c$  = speed of sound in the fluid (input as SONC on **MP** command)  
 $\ddot{P}$  = second derivative of pressure with respect to time  
 $\Omega^+$  = unbounded region occupied by the fluid

In addition to equation (14.130–1), the following Sommerfeld radiation condition (which simply states that the waves generated within the fluid are outgoing) needs to be satisfied at infinity:

$$\lim_{r \rightarrow \infty} r \frac{d-1}{2} \left( P_r + \frac{1}{c} \dot{P} \right) = 0. \quad (14.130-2)$$

where:  $r$  = distance from the origin  
 $P_r$  = pressure derivative along the radial direction  
 $d$  = dimensionality of the problem (i.e.,  $d=3$  or  $d=2$  if  $\Omega^+$  is 3D or 2D, respectively)

A primary difficulty associated with the use of finite elements for the modeling of the infinite medium stems precisely from the need to satisfy the Sommerfeld radiation condition, equation (14.130–2). A typical approach for tackling the difficulty consists of truncating the unbounded domain  $\Omega^+$  by the introduction of an absorbing (artificial) boundary  $\Gamma_a$  at some distance from the structure.



The equation of motion (14.130–1) is then solved in the annular region  $\Omega^f$  which is bounded by the fluid–structure interface  $\Gamma$  and the absorbing boundary  $\Gamma_a$ . In order, however, for the resulting problem in  $\Omega^f$  to be well–posed, an appropriate condition needs to be specified on  $\Gamma_a$ . Towards this end, the following second–order conditions are used (Kallivokas et al.(218)) on  $\Gamma_a$ :

**In two dimensions:**

$$\dot{P}_n + \gamma P_n = -\frac{1}{c} \ddot{P} + \left( \frac{1}{2} \kappa - \frac{\gamma}{c} \right) \dot{P} + \frac{1}{2} c P_{\lambda\lambda} + \left( \frac{1}{8} \kappa^2 c + \frac{1}{2} \kappa \gamma \right) P \quad (14.130-3)$$

where:  $n$  = outward normal to  $\Gamma_a$

- $P_n$  = pressure derivative in the normal direction  
 $P_{\lambda\lambda}$  = pressure derivative along  $\Gamma_\alpha$   
 $\kappa$  = curvature of  $\Gamma_a$   
 $\gamma$  = stability parameter

**In three dimensions:**

$$\begin{aligned} \dot{P}_n + \gamma P_n = & -\frac{1}{c}\dot{P} + \left(H - \frac{\gamma}{c}\right)\dot{P} \\ & + H\gamma P + \frac{c}{2\sqrt{EG}} \left[ \left(\sqrt{\frac{G}{E}} P_u\right)_u + \left(\sqrt{\frac{G}{E}} P_v\right)_v \right] + \frac{c}{2}(H^2 - K) P \end{aligned} \quad (14.130-4)$$

- where:
- $n$  = outward normal
  - $u$  and  $v$  = orthogonal curvilinear surface coordinates (e.g., the meridional and polar angles in spherical coordinates)
  - $P_u, P_v$  = pressure derivatives in the  $\Gamma_a$  surface directions
  - $H$  and  $K$  = mean and Gaussian curvature, respectively
  - $E$  and  $G$  = usual coefficients of the first fundamental form

## 14.130-2 Finite Element Discretization

Following a Galerkin based procedure, equation (14.130-1) is multiplied by a virtual quantity  $\delta P$  and integrated over the annular domain  $\Omega^f$ . By using the divergence theorem on the resulting equation it can be shown that:

$$\frac{1}{c^2} \int_{\Omega^f} \delta P \ddot{P} d\Omega^f + \int_{\Omega^f} \nabla \delta P \cdot \nabla P d\Omega^f - \int_{\Gamma_a} \delta P P_n d\Gamma_a = - \int_{\Gamma} \delta P P_n d\Gamma \quad (14.130-5)$$

Upon discretization of equation (14.130-5), the first term on the left hand side will yield the mass matrix of the fluid while the second term will yield the stiffness matrix.

Next, the following finite element approximations for quantities on the absorbing boundary  $\Gamma_a$  placed at a radius  $R$  and their virtual counterparts are introduced:

$$P(x, t) = \mathbf{N}_1^T(x) \mathbf{P}(t) \quad , \quad q^{(1)}(x, t) = \mathbf{N}_2(x) q^{(1)}(t) \quad , \quad q^{(2)}(x, t) = \mathbf{N}_3^T(x) q^{(2)}(t) \quad (14.130-6)$$

$$\delta P(x) = \delta \mathbf{P}^T \mathbf{N}_1(x) \quad , \quad \delta q^{(1)}(x) = \delta q^{(1)T} \mathbf{N}_2(x) \quad , \quad \delta q^{(2)}(x) = \delta q^{(2)T} \mathbf{N}_3(x) \quad (14.130-7)$$

where:  $\mathbf{N}_1, \mathbf{N}_2, \mathbf{N}_3$  = vectors of shape functions (=  $\{\mathbf{N}_1\}, \{\mathbf{N}_2\}, \{\mathbf{N}_3\}$ )

$P, q^{(1)}, q^{(2)}$  = unknown nodal values ( $P$  is output as degree of freedom PRES.  $q^{(1)}$  and  $q^{(2)}$  are solved for but not output).

Furthermore, the shape functions in equations (14.130–6) and (14.130–7) are set to:

$$\mathbf{N}_1 = \mathbf{N}_2 = \mathbf{N}_3 = \mathbf{N} \quad (14.130-8)$$

The element stiffness and damping matrices reduce to:

**For two dimensional case:**

$$[\mathbf{K}_a^{2D}] = \frac{1}{8R} \begin{bmatrix} 4 \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_c & 4R^2 \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_c & - \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_c \\ 4R^2 \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_e & -4R^2 \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_e & 0 \\ - \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_e & 0 & \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_e \end{bmatrix} \quad (14.130-9)$$

$$[\mathbf{C}_a^{2D}] = \frac{1}{8c} \begin{bmatrix} 8 \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_e & 0 & 0 \\ 0 & -4R^2 \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_e & 0 \\ 0 & 0 & \int_{\Gamma_a^c} \mathbf{N} \mathbf{N}^T d\lambda_e \end{bmatrix} \quad (14.130-10)$$

where:  $d\lambda_e$  = arc-length differential

These matrices are 6 x 6 in size, having 2 nodes per element with 3 degrees of freedom per node ( $P, q^{(1)}, q^{(2)}$ ).

**For three dimensional case:**

$$[K_a^{3D}] = \frac{1}{2R} \begin{bmatrix} 2 \int_{\Gamma_a^e} \mathbf{N} \mathbf{N}^T dA_e & R^2 \int_{\Gamma_a^e} \bar{\nabla}^s \mathbf{N} \cdot \bar{\nabla}^s \mathbf{N}^T dA_e \\ R^2 \int_{\Gamma_a^e} \bar{\nabla}^s \mathbf{N}^T \cdot \bar{\nabla}^s \mathbf{N} dA_e & -R^2 \int_{\Gamma_a^e} \bar{\nabla}^s \mathbf{N} \cdot \bar{\nabla}^s \mathbf{N}^T dA_e \end{bmatrix} \quad (14.130-11)$$

$$[C_a^{3D}] = \frac{1}{2c} \begin{bmatrix} 2 \int_{\Gamma_a^e} \mathbf{N} \mathbf{N}^T dA_e & 0 \\ 0 & -R^2 \int_{\Gamma_a^e} \bar{\nabla}^s \mathbf{N} \cdot \bar{\nabla}^s \mathbf{N}^T dA_e \end{bmatrix} \quad (14.130-12)$$

where:  $dA_e$  = area differential

These matrices are 8 x 8 in size, having 4 nodes per element with 2 degrees of freedom per node (P, q) (Barry et al. (217)).

**For axisymmetric case:**

$$[K_a^{2Da}] = \frac{\pi}{R} \begin{bmatrix} 2 \int_{\Gamma_a^e} \mathbf{N} \mathbf{N}^T x d\lambda_c & R^2 \int_{\Gamma_a^e} \mathbf{N} \mathbf{N}^T x d\lambda_c \\ R^2 \int_{\Gamma_a^e} \mathbf{N}^T \mathbf{N} x d\lambda_e & -R^2 \int_{\Gamma_a^e} \mathbf{N} \mathbf{N}^T x d\lambda_e \end{bmatrix} \quad (14.130-13)$$

$$[C_a^{2Da}] = \frac{\pi}{c} \begin{bmatrix} 2 \int_{\Gamma_a^e} \mathbf{N} \mathbf{N}^T x d\lambda_c & 0 \\ 0 & -R^2 \int_{\Gamma_a^e} \mathbf{N} \mathbf{N}^T x d\lambda_e \end{bmatrix} \quad (14.130-14)$$

where:  $x$  = radius

These matrices are 4 x 4 in size having 2 nodes per element with 2 degrees of freedom per node (P, q).

