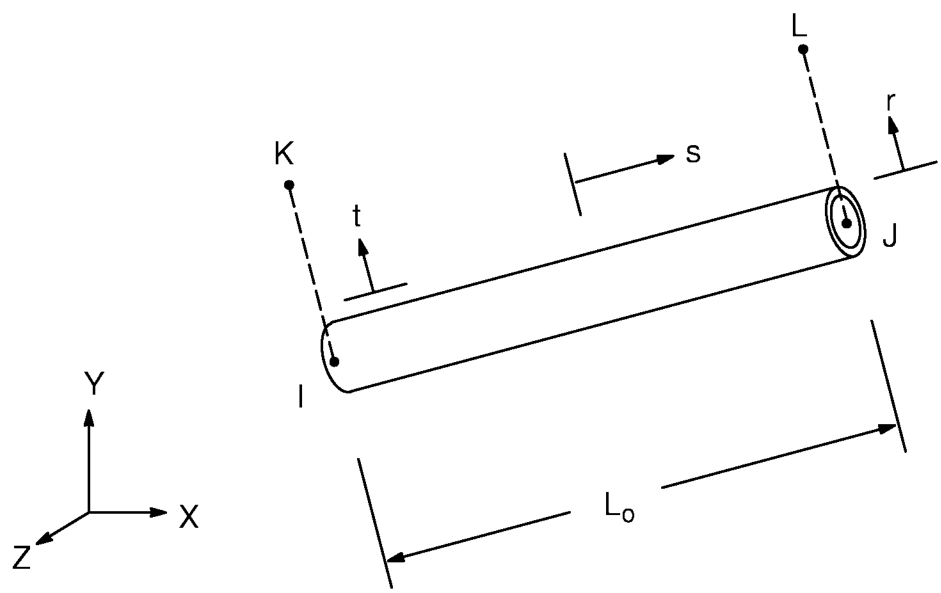
14.116 FLUID116 — Coupled Thermal–Fluid Pipe



Matrix or Vector	Geometry	Shape Functions	Integration Points
Thermal Conductivity Matrix	Between nodes I and J	Equation (12.2.1–20)	None
	Convection between nodes I and K and between nodes J and L (optional)	None	None
Pressure Conductivity Matrix	Between nodes I and J	Equation (12.2.1–19)	None

Matrix or Vector	Geometry	Shape Functions	Integration Points
Specific Heat Matrix	Equation (12.2.1–20)		None
Heat Generation Load Vector	Equation (12.2.1–20)		None

14.116.1 Assumptions and Restrictions

Transient pressure and compressibility effects are also not included.

14.116.2 Combined Equations

The thermal and pressure aspects of the problem have been combined into one element having two different types of working variables: temperatures and pressures. The equilibrium equations for one element have the form of:

$$N_c \begin{bmatrix} [C^t] & [0] \\ [0] & [0] \end{bmatrix} \begin{Bmatrix} \{\dot{T}\} \\ \{0\} \end{Bmatrix} + N_c \begin{bmatrix} [K^t] & [0] \\ [0] & [K^p] \end{bmatrix} \begin{Bmatrix} \{T\} \\ \{P\} \end{Bmatrix} = \begin{Bmatrix} \{Q\} \\ \{w\} \end{Bmatrix} + N_c \begin{Bmatrix} \{Q^g\} \\ \{H\} \end{Bmatrix} \quad (14.116\text{–}1)$$

where:

$[C^t]$ = specific heat matrix for one channel

$\{T\}$ = nodal temperature vector

$\{\dot{T}\}$ = vector of variations of nodal temperature with respect to time

$\{P\}$ = nodal pressure vector

$[K^t]$ = thermal conductivity matrix for one channel (includes effects of convection and mass transport)

$[K^p]$ = pressure conductivity matrix for one channel

$\{Q\}$ = nodal heat flow vector (input quantity HEAT on **F** command)

$\{w\}$ = nodal fluid flow vector (input quantity FLOW on **F** command)

$\{Q^g\}$ = internal heat generation vector for one channel

$\{H\}$ = gravity and pumping effects vector for one channel

N_c = number of parallel flow channels (input quantity N_c on **R** command)

14.116.3 Thermal Matrix Definitions

Specific Heat Matrix

The specific heat matrix is a diagonal matrix with each term being the sum of the corresponding row of a consistent specific heat matrix:

$$[C^t] = A_c \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (14.116\text{–}2)$$

where: $A_c = \frac{\rho_u C_p A L_o}{2}$

ρ_u = effective density

$$= \begin{cases} \rho & \text{if } R_{gas} = 0.0 \\ \text{or} & \\ \frac{P}{R_{gas}\,T_{abs}} \quad \text{(ideal gas law)} & \text{if } R_{gas} \neq 0.0 \end{cases}$$

ρ = mass density (input as DENS on **MP** command

P = pressure (average of first two nodes)

T_{abs} = T + TOFFST = absolute temperature

T = temperature (average of first two nodes)

TOFFST = offset temperature (input on **TOFFST** command)

C_p = specific heat (input as C on **MP** command)

A = flow cross–sectional area (input quantity A on **R** command)

L_o = length of member (distance between nodes I and J)

R_{gas} = gas constant (input quantity R_{gas} on **R** command)

Thermal Conductivity Matrix

The thermal conductivity matrix is given by:

$$[K^t] = \begin{bmatrix} B_1 + B_2 - B_4 & -B_1 + B_4 & -B_2 & 0 \\ -B_1 - B_5 & B_1 + B_3 + B_5 & 0 & -B_3 \\ -B_2 & 0 & B_2 & 0 \\ 0 & -B_3 & 0 & B_3 \end{bmatrix} \qquad (14.116\text{–}3)$$

where: $B_1 = \frac{AK_s}{\ell}$

K_s = thermal conductivity (input as KXX on **MP** command)

B_2 = h A_I

h = film coefficient (defined below)

A_I = lateral area of pipe associated with end I (input quantity $(A_n)_I$ on **R** command) (defaults to $\frac{\pi D \ell}{2}$)

B_3 = h A_J

A_j = lateral area of pipe associated with end J (input quantity $(A_n)_J$ on **R** command) (defaults to $\frac{\pi D \ell}{2}$)

D = hydraulic diameter (input quantity D on **R** Command)

$$B_4 = \begin{cases} wC_p \text{ if flow is from node J to node I} \\ 0 \text{ if flow is from node I to node J} \end{cases}$$

$$B_5 = \begin{cases} wC_p \text{ if flow is from node I to node J} \\ 0 \text{ if flow is from node J to node I} \end{cases}$$

w = mass fluid flow rate in the element

w may be determined by the program or may be input by the user:

$$w = \begin{cases} \text{computed from previous iteration} & \text{if pressure is a degree of freedom} \\ \text{or} & \\ \text{input as VAL1 on } \textbf{SFE,,,}\text{HFLUX command} & \text{if pressure is not a degree of freedom} \end{cases} \quad (14.116\text{–}4)$$

The above definitions of B_4 and B_5, as used by equation NO TAG, cause the energy change due to mass transport to be lumped at the outlet node.

The film coefficient h is defined as:

$$h = \begin{cases} \text{input as HF on } \textbf{MP} \text{ command} & \text{if KEYOPT(4) = 0} \\ \text{or} & \\ \frac{Nu\ K_s}{D} & \text{if KEYOPT(4) = 1} \\ \text{or} & \\ \text{defined by TB, HFLM table} & \text{if KEYOPT(4) = 2, 3, or 4} \\ \text{or} & \\ \text{defined by user programmable feature User116Hf} & \text{if KEYOPT(4) = 5} \end{cases} \quad (14.116\text{–}5)$$

Nu, the Nusselt number, is defined for KEYOPT(4) = 1 as:

$$Nu = N_1 + N_2 \, Re^{N_3} \, Pr^{N_4} \tag{14.116–6}$$

where:
N_1 to N_4 = input on **R** commands

$Re = \frac{w \, D}{\mu A}$

= Reynold's number

μ = viscosity (input as VISC on **MP** command)

$Pr = \frac{C_p \, \mu}{K_s}$

= Prandtl number

A common usage of equation (14.116–6) is the Dittus–Boelter correlation for fully developed turbulent flow in smooth tubes (Holman(55)):

$$Nu = 0.023 \, Re^{0.8} \, Pr^{a} \tag{14.116–7}$$

where:

$$a = \begin{cases} 0.4 \text{ for heating} \\ 0.3 \text{ for cooling} \end{cases}$$

Heat Generation Load Vector

The internal heat generation load vector is due to both average heating effects and viscous damping:

$$\{Q^g\} = \begin{Bmatrix} Q_n \\ Q_n \\ 0 \\ 0 \end{Bmatrix} \tag{14.116–8}$$

where:

$Q_n = \frac{L_o}{2} \left(A\dddot{q} + \pi \, V_{DF} \, C_{ver} \, F \, \mu \, v^2 \right)$

$\dddot{q}$ = internal heat generation rate per unit volume

= input on **BF** or **BFE** commands

V_{DF} = viscous damping multiplier (input on **RMORE** command)

C_{ver} = units conversion factor (input on **RMORE** command)

F = flow type factor

$$= \begin{cases} 8.0 & \text{if} \quad Re \le 2500.0 \\ 0.21420 & \text{if} \quad Re > 2500.0 \end{cases}$$

v = average velocity

The expression for the viscous damping part of Q_n is based on fully developed laminar flow.

14.116.4 Fluid Equations

Bernoulli's equation is:

$$Z_I + \frac{P_I}{\gamma} + \frac{v_I^2}{2g} + \frac{P_{PMP}}{\gamma} = Z_J + \frac{P_J}{\gamma} + \frac{v_J^2}{2g} + C_L \frac{v_a^2}{2g} \quad (14.116\text{–}9)$$

where:

- Z = coordinate in the negative acceleration direction
- P = pressure
- γ = ρg
- g = acceleration of gravity
- v = velocity
- P_{PMP} = pump pressure (input as P_p on **R** command)
- C_L = loss coefficient

The loss coefficient is defined as:

$$C_L = \frac{f\ell}{D} + \beta\ell \quad (14.116\text{–}10)$$

where:

$$\beta = \begin{cases} \dfrac{f\ell_a}{D\ell} & \text{if KEYOPT(8)} = 0 \\ \text{or} & \\ \dfrac{k}{\ell} & \text{if KEYOPT(8)} = 1 \end{cases}$$

- = extra flow loss factor
- ℓ_a = additional length to account for extra flow losses (input as L_a on **R** command)
- k = loss coefficient for typical fittings (input as K on **R** command)
- f = Moody friction factor, defined below:

For the first iteration of the first load step,

$$f = \begin{cases} f_m & \text{if } f_m \neq 0.0 \\ 1.0 & \text{if } f_m = 0.0 \end{cases} \quad (14.116\text{–}11)$$

where: f_m = input as MU on **MP** command

For all subsequent iterations

$$f = \begin{cases} f_x & \text{if KEYOPT(7)} = 0 \\ f_m & \text{if KEYOPT(7)} = 1 \\ \text{defined by TB, FLOW} & \text{if KEYOPT(7)} = 2, 3 \end{cases} \tag{14.116–12}$$

The smooth pipe empirical correlation is:

$$f_x = \begin{cases} \frac{64}{Re} & 0 < Re \leq 2500 \\ \text{or} & \\ \frac{0.316}{(Re)^{1/4}} & 2500 < Re \end{cases} \tag{14.116–13}$$

Bernoulli's equation (14.116–9) may be simplified for this element, since the cross–sectional area of the pipe does not change. Therefore, continuity requires all velocities not to vary along the length. Hence $v_1 = v_2 = v_a$, so that Bernoulli's equation (14.116–9) reduces to:

$$Z_I - Z_J + \frac{P_I - P_J}{\gamma} + \frac{P_{PMP}}{\gamma} = C_L \frac{v^2}{2g} \tag{14.116–14}$$

Writing equation (14.116–14) in terms of mass flow rate ($w = \rho A v$), and rearranging terms to match the second half of equation (14.116–1),

$$\frac{2\rho A^2}{C_L} (P_I - P_J) = w^2 + \frac{2g\rho^2 A^2}{C_L} \left(-Z_I + Z_J - \frac{P_{PMP}}{\gamma}\right) \tag{14.116–15}$$

Since the pressure drop ($P_I - P_J$) is not linearly related to the flow (w), a nonlinear solution will be required. As the w term may not be squared in the solution, the square root of all terms is taken in a heuristic way:

$$A \sqrt{\frac{2\rho}{C_L}} \sqrt{P_I - P_J} = w + A \sqrt{\frac{2\rho}{C_L}} \left((-Z_I + Z_J)\,\rho g - P_{PMP}\right) \tag{14.116–16}$$

Defining:

$$B_c = A \sqrt{\frac{2\rho}{C_L}} \tag{14.116–17}$$

and

$$P_L = (-Z_I + Z_J)\rho g - P_{PMP} \tag{14.116–18}$$

equation (14.116–16) reduces to:

$$B_c \sqrt{P_I - P_J} = w + B_c P_L \tag{14.116–19}$$

Hence, the pressure conductivity matrix is based on the term $\frac{B_c}{\sqrt{P_I - P_J}}$ and the pressure (gravity and pumping) load vector is based on the term $B_c P_L$.

Two further points:

1. B_c is generalized as:

$$B_c = \begin{cases} A\sqrt{\frac{2\rho}{C_L}} & \text{if KEYOPT(6) = 0} \\ \text{input as C on \textbf{R} command,} & \text{if KEYOPT(6) = 1} \\ \text{defined by TB,FCON} & \text{if KEYOPT(6) = 2 or 3} \\ \text{defined by user programmable feature, User116Cond} & \text{if KEYOPT(6) = 4} \end{cases} \tag{14.116–20}$$

2. $(-Z_I + Z_J)g$ is generalized as:

$$(-Z_I + Z_J)g = \{\Delta x\}^T\{a_t\} \tag{14.116–21}$$

where:

$\{\Delta x\}$ = vector from node I to node J

$\{a_t\}$ = translational acceleration vector which includes effects of angular velocities (see section 15.1)