14.44   BEAM44 — 3–D Elastic Tapered Unsymmetrical Beam
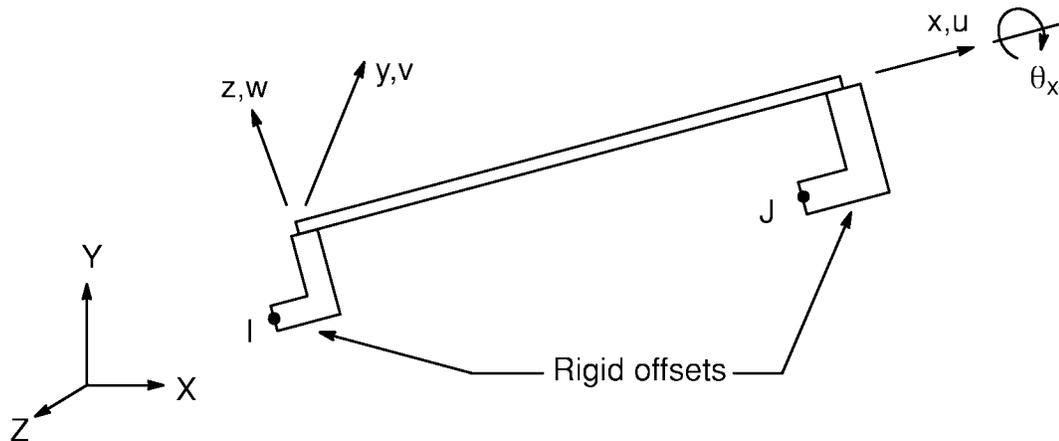
Matrix or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equations (12.2.2–1), (12.2.2–2), (12.2.2–3) and (12.2.2–4)	None
Foundation Stiffness Matrix	Same as stiffness matrix	None
Mass Matrix	If consistent mass matrix option is used (KEYOPT(2) = 0), same as stiffness matrix. If reduced mass matrix option is used (KEYOPT(2) = 1), equations (12.2.1–1), (12.2.1–2) and (12.2.1–3)	None
Stress Stiffness Matrix	Equations (12.2.2–2) and (12.2.2–3)	None
Load Vector (Pressures and Temperatures)	Equations (12.2.2–1), (12.2.2–2), and (12.2.2–3)	None

Load Type	Distribution
Element Temperature	Bilinear across cross–section, linear along length
Nodal Temperature	Constant across cross–section, linear along length
Pressure	Linear along length

14.44.1   Other Applicable Sections

This element is an extension of BEAM4, so that the basic element formulation as well as the local to global matrix conversion logic is described in Section 14.4.

14.44.2   Assumptions and Restrictions

1. Normals before deformation remain straight and normal after deformation.

2. Offsets, if any, are assumed to be completely rigid.

3. If both offsets and also angular velocities or angular accelerations (input on **OMEGA**, **DOMEGA**, **CGOMGA**, or **DCGOMG** commands) are used, the radius used in the inertial force calculations does not account for the offsets.

4. Foundation stiffness effects are applied on the flexible length (i.e., before offsets are used).

5. Shear deflection effects are not included in the mass matrix, as they are for BEAM4.

6. Thermal bending assumes an (average) uniform thickness.

14.44.3   Tapered Geometry

When a tapered geometry is input, the program has no "correct" form to follow as the program does not know the shape of the cross–section. The supplied thicknesses are used only for thermal bending and stress evaluation. Consider the case of a beam with an area of 1.0 at one end and 4.0 at the other. Assuming all tapers are straight, the small end is a square, the large end is a 1.0 × 4.0 rectangular, and the midpoint of the beam would then have an area of 2.50. But if the large end is also square (2.0 × 2.0), the midpoint area would then be 2.25. Thus, there is no unique solution. All effects of approximations are reduced by ensuring that the ratios of the section properties are as close to 1.0 as possible. The discussion below indicates what is done for this element.

The stiffness matrix is the same as for BEAM4 (equation (14.4–1)), except that an averaged area is used:

$$A_{AV} = \left(A_1 + \sqrt{A_1 A_2} + A_2\right) / 3 \tag{14.44-1}$$

and all three moments of inertia use averages of the form:

$$I_{AV} = \left(I_1 + \sqrt[4]{I_1{}^3 I_2} + \sqrt{I_1 I_2} + \sqrt[4]{I_1 I_2{}^3} + I_2\right) / 5 \tag{14.44-2}$$

The mass matrix is also the same as for BEAM4 (equation (14.4-2)), except the upper left quadrant uses section properties only from end I, the lower right quadrant uses section properties only from end J, and the other two quadrants use averaged values. For example, assuming no prestrain or added mass, the axial mass terms would be ϱA_1 L/3 for end I, ϱA_2 L/3 for end J, and $\varrho(A_1 + A_2)$ L/12 for both off–diagonal terms. Thus, the total mass of the element is: $\varrho(A_1 + A_2)$ L/2.

The stress stiffness matrix assumes a constant area as determined in equation (14.44-1).

Finally, the thermal load vector uses average thicknesses.

14.44.4 Shear Center Effects

The shear center effects affect only the torsional terms (M_x, θ_x). The rotation matrix [R^s] (used below) is:

$$[R^s] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & C_1 & 0 & 0 \\ 0 & 0 & 0 & C_2 & 1 & 0 \\ 0 & 0 & 0 & C_3 & 0 & 1 \end{bmatrix} \tag{14.44-3}$$

where:

$$C_1 = \frac{L_{SC}}{L_G}$$

$$C_2 = -\frac{\Delta_y^s \, L_{SC}}{L_{SB} \, L_G}$$

$$C_3 = -\frac{\Delta_z^s}{L_{SB}}$$

$$L_{SC} = \sqrt{(L_G)^2 + (\Delta_y^s)^2 + (\Delta_z^s)^2}$$

$$L_{SB} = \sqrt{(L_G)^2 + (\Delta_y^s)^2}$$

$$\Delta_y^s = \Delta_{y2}^s - \Delta_{y1}^s$$

$$\Delta_z^s = \Delta_{z2}^s - \Delta_{z1}^s$$

Δ_{y2}^s = input quantity DYSC2 on **RMORE** command

L_G = actual flexible length, as shown in Figure 14.44–1

Note that only rotation about the shear centerline (θ_x) is affected. The shear center translations at node I are accounted for by:

$$\left[T_I^s\right] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\Delta_{z1}{}^s & 0 & 0 \\ 0 & 0 & 1 & \Delta_{y1}{}^s & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{14.44–4}$$

A similar matrix $\left[T_J^s\right]$ is defined at node J based on Δ_{y2}^s and Δ_{z2}^s. These matrices are then combined to generate the $[S_c]$ matrix:

$$[S_c] = \begin{bmatrix} \left[R^s T_I^s\right] & [0] \\ [0] & \left[R^s T_J^s\right] \end{bmatrix} \tag{14.44–5}$$

This combination of [R] and [T] results because shear center offsets are measured in the element coordinate system ($x^e \, y^e \, z^e$ in Figure 14.44–1). The element matrices are then transformed by
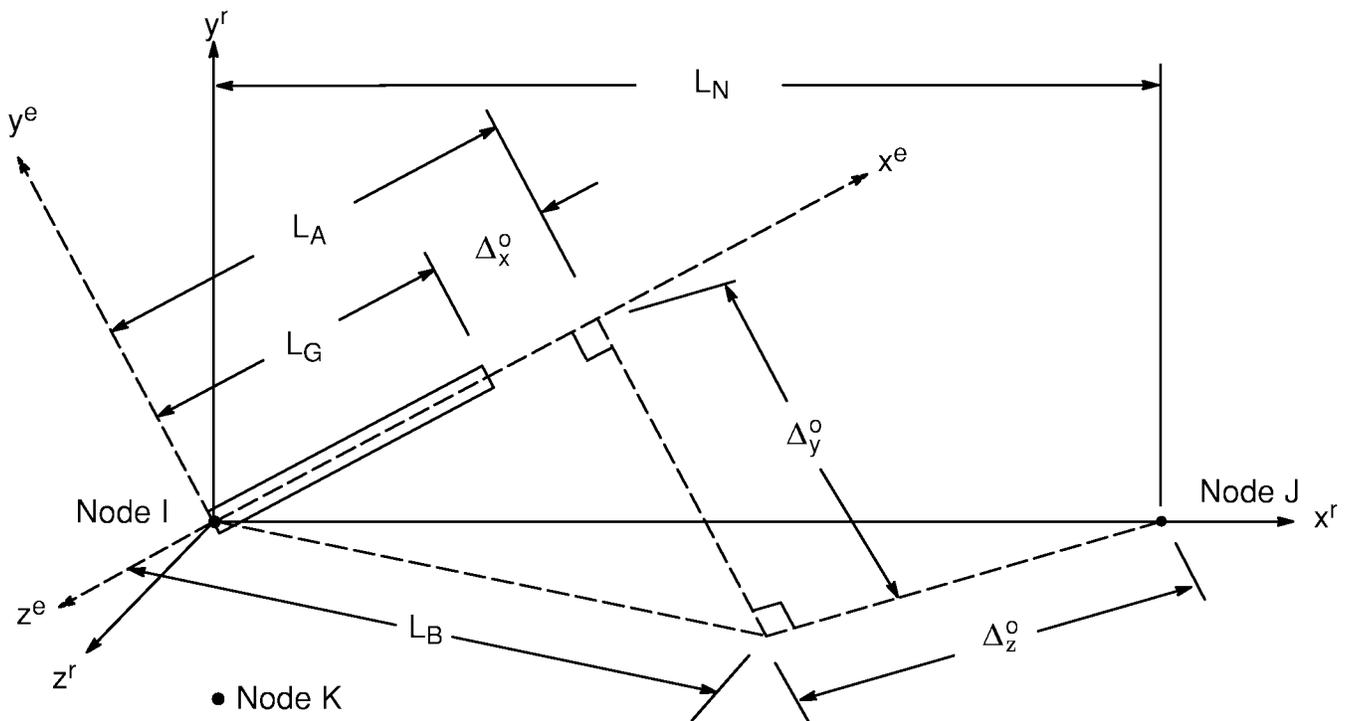
$$[K_\ell'] = [S_c]^T [K_\ell][S_c] \tag{14.44–6}$$

$$[S_\ell'] = [S_c]^T [S_\ell][S_c] \tag{14.44–7}$$

$$\{F_\ell'\} = [S_c]^T \{F_\ell\} \tag{14.44–8}$$

where: $\left[K_\ell\right]$ = element stiffness matrix in element (centroidal) coordinate system, similar to equation (14.4–1)

$\left[S_\ell\right]$ = element stress stiffness matrix in element (centroidal) coordinate system

$\{F_\ell\}$ = element load vector in element (centroidal) coordinate system, similar to equation (14.4–4).

1. Nodes I and J define the x^r axis
2. Nodes I, J, and K define the plane of the z^r axis
3. The z^e axis, as well as the Δ_z^o offset, lie parallel to the x^r–z^r plane
4. L_G is the flexible length

Figure 14.44–1 Offset Geometry

14.44.5   Offset at the Ends of the Member

It is convenient to define

$$\Delta_x^o = \Delta_{x2} - \Delta_{x1}$$

$$\Delta_y^o = \Delta_{y2} - \Delta_{y1}$$

$$\Delta_z^o = \Delta_{z2} - \Delta_{z1}$$

where: Δ_{x2} =   input quantity DX2 on **RMORE** command

These definitions of Δ_i^o may be thought of as simply setting the offsets at node I to zero and setting the differential offset to the offset at node J, as shown in Figure 14.44–1. The rotation matrix [R^o] implied by the offsets is defined by:

$$\lfloor u_x^e \; u_y^e \; u_z^e \; \theta_x^e \; \theta_y^e \; \theta_z^e \rfloor^T = [R^o] \lfloor u_x^r \; u_y^r \; u_z^r \; \theta_x^r \; \theta_y^r \; \theta_z^r \rfloor^T \tag{14.44-9}$$

where: u_x^e, u_y^e, etc.   =   are in the element coordinate system

           u_x^r, u_y^r, etc.   =   are in the reference coordinate system defined by the nodes

$$[R^o] \;=\; \begin{bmatrix} [r^o] & [0] \\ [0] & [r^o] \end{bmatrix}$$

$$[r^o] \;=\; \begin{bmatrix} \dfrac{L_\Lambda}{L_N} & \dfrac{\Delta_y^o}{L_B} & \dfrac{L_\Lambda \, \Delta_z^o}{L_N \, L_B} \\[2ex] -\dfrac{\Delta_y^o}{L_N} & \dfrac{L_\Lambda}{L_B} & -\dfrac{\Delta_y^o \, \Delta_z^o}{L_N \, L_B} \\[2ex] -\dfrac{\Delta_z^o}{L_N} & 0 & \dfrac{L_B}{L_N} \end{bmatrix}$$

To account for the translation of forces and moments due to offsets at node I, matrix $\left[T_i^o \right]$ is defined using Figure 14.44–2.
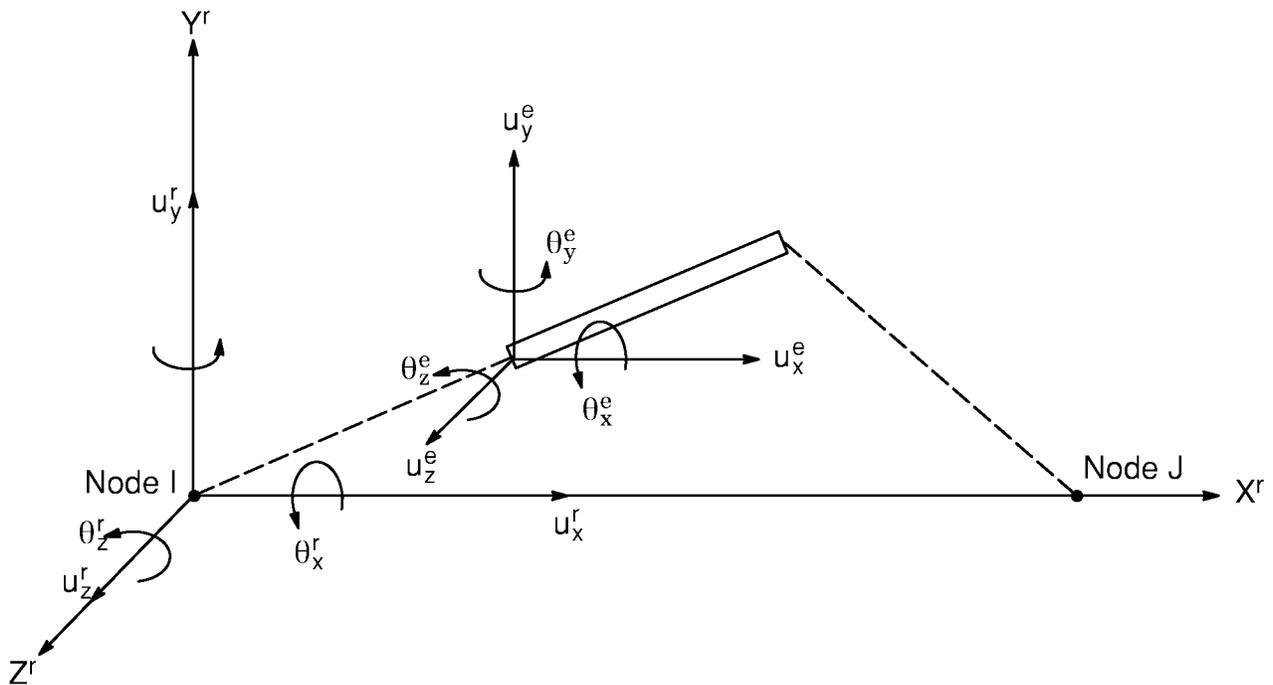

Figure 14.44–2 Translation of Axes

The two systems are related by:

$$\lfloor u_x^e \quad u_y^e \quad u_z^e \quad \theta_x^e \quad \theta_y^e \quad \theta_z^e \rfloor^T = \left[T_I^o \right] \lfloor u_x^r \quad u_y^r \quad u_z^r \quad \theta_x^r \quad \theta_y^r \quad \theta_z^r \rfloor^T \qquad (14.44\text{--}10)$$

where:

$$\left[T_I^o \right] = \begin{bmatrix} 1 & 0 & 0 & 0 & \Delta_{z1} & -\Delta_{y1} \\ 0 & 1 & 0 & -\Delta_{z1} & 0 & \Delta_{x1} \\ 0 & 0 & 1 & \Delta_{y1} & -\Delta_{x1} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

A similar matrix $\left[T_J^o \right]$ is defined at node J, based on Δ_{x2}, Δ_{y2}, and Δ_{z2}. These matrices are then combined to generate the $[O_F]$ matrix:

$$[O_F] = \begin{bmatrix} \left[T_I^o \right][R^o] & [0] \\ [0] & \left[T_J^o \right][R^o] \end{bmatrix} \qquad (14.44\text{--}11)$$

The basis for the above transformations is taken from Hall and Woodhead(15). The element matrices are then transformed again by:

$$[K_\ell''] = [O_F]^T [K_\ell'][O_F] \qquad (14.44\text{--}12)$$

$$[S_\ell''] = [O_F]^T [S_\ell'][O_F] \qquad (14.44\text{--}13)$$

$$[M_\ell'] = [O_F]^T [M_\ell][O_F] \qquad (14.44\text{--}14)$$

$$\{F_\ell''\} = [O_F]^T \{F_\ell'\} \qquad (14.44\text{--}15)$$

where: $\quad [M_\ell] \quad = \quad$ element mass matrix in element (centroidal) coordinate system, similar to equation (14.4–2).

14.44.6   End Moment Release

End moment release (or end rotational stiffness release) logic is activated if either KEYOPT(7) or KEYOPT(8) > 0. The release logic is analogous to that discussed in

Section 17.6, with the dropped rotational DOF represented by the slave DOF. The processing of the matrices may be symbolized by:

$$[K_\ell^{''}] => [K_\ell^{''}] \quad \text{using static condensation (equation (17.6–8))} \quad (14.44–16)$$

$$[S_\ell^{''}] => [S_\ell^{''}] \begin{cases} \text{using Guyan reduction (equation (17.6–19)) if} \\ \textit{Type} = \text{BUCKLE on the } \textbf{ANTYPE} \text{ command} \\ \text{(case of linear buckling)} \\ \text{using static condensation (equation (17.6–8))} \\ \text{after being combined with } [K_\ell^{''}] \text{ if } \textit{Type} \neq \text{BUCKLE} \\ \text{on the } \textbf{ANTYPE} \text{ command (cases other than linear} \\ \text{buckling)} \end{cases} \quad (14.44–17)$$

$$[M_\ell^{'}] => [M_\ell^{'}] \quad \text{using Guyan reduction (equation (17.6–19))} \quad (14.44–18)$$

$$\{F_\ell^{''}\} => \{F_\ell^{''}\} \quad \text{using static condensation (equation (17.6–9))} \quad (14.44–19)$$

14.44.7   Local to Global Conversion

The generation of the local to global transformation matrix $[T_R]$ is discussed in Section 14.4. Thus, the final matrix conversions are:

$$[K_e] = [T_R]^T [K_\ell^{''}][T_R] \tag{14.44–20}$$

$$[S_e] = [T_R]^T [S_\ell^{''}][T_R] \tag{14.44–21}$$

$$[M_e] = [T_R]^T [M_\ell^{'}][T_R] \tag{14.44–22}$$

$$\{F_e\} = [T_R]^T \{F_\ell^{''}\} \tag{14.44–23}$$

14.44.8   Stress Calculations

The axial stresses are computed analogously to BEAM4. The maximum stress at cross section i is computed by:

$$\sigma_i^{max} = \text{maximum of} \begin{cases} \sigma_i^{dir} + \sigma_{zt,i}^{bnd} + \sigma_{yt,i}^{bnd} \\ \sigma_i^{dir} + \sigma_{zt,i}^{bnd} + \sigma_{yb,i}^{bnd} \\ \sigma_i^{dir} + \sigma_{zb,i}^{bnd} + \sigma_{yb,i}^{bnd} \\ \sigma_i^{dir} + \sigma_{zb,i}^{bnd} + \sigma_{yt,i}^{bnd} \end{cases}$$

(14.44–24)

where:

σ^{dir}	=	output quantity SDIR
σ_{yt}^{bnd}	=	output quantity SBYT
σ_{yb}^{bnd}	=	output quantity SBYB
σ_{zt}^{bnd}	=	output quantity SBZT
σ_{zb}^{bnd}	=	output quantity SBZB

The minimum stress is analogously defined..

The shear stresses are computed as:

$$\tau_L^y = \frac{F^y}{A_s^y}$$

(14.44–25)

$$\tau_L^z = \frac{F^z}{A_s^z}$$

(14.44–26)

where:

τ_L^y, τ_L^z	=	transverse shear stress (output quantities SXY, SXZ)
F^y, F^z	=	transverse shear forces
A_s^y, A_s^z	=	transverse shear areas (input quantities ARESZ1, etc. on **RMORE** command)

and

$$\tau_T = M_x C$$

(14.44–27)

where:

τ_T	=	torsional shear stress (output quantity SYZ)
M_x	=	torsion moment
C	=	user–supplied constant (input quantities TSF1 and TSF2 on **RMORE** command)