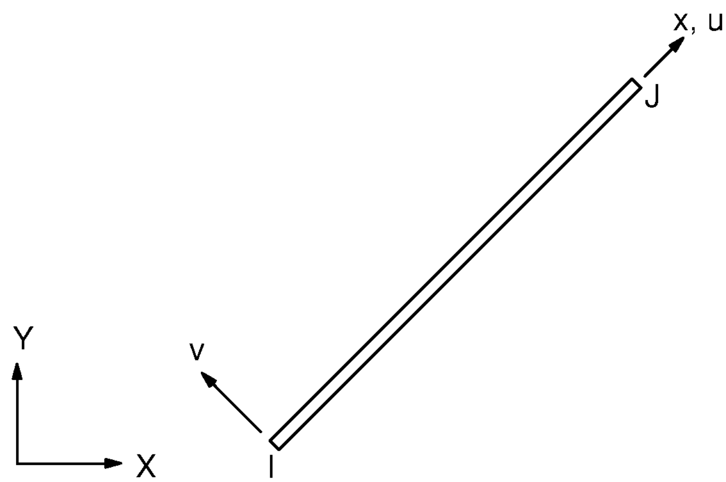
14.3 BEAM3 — 2–D Elastic Beam



Matrix Or Vector	Shape Functions	Integration Points
Stiffness Matrix	Equation (12.1.2–1) and (12.1.2–2)	None
Mass Matrix	Same as stiffness matrix	None
Stress Stiffness Matrix	Equation (12.1.2–2)	None
Thermal and Pressure Load Vector	Same as stiffness matrix	None

Load Type	Distribution
Element Temperature	Linear thru thickness and along length
Nodal Temperature	Constant thru thickness, linear along length
Pressure	Linear along length

14.3.1 Element Matrices and Load Vectors

The element stiffness matrix in element coordinates is (Przemieniecki(28)):

$$[K_\ell] = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ 0 & \frac{12EI}{L^3(1+\phi)} & \frac{6EI}{L^2(1+\phi)} & 0 & -\frac{12EI}{L^3(1+\phi)} & \frac{6EI}{L^2(1+\phi)} \\ 0 & \frac{6EI}{L^2(1+\phi)} & \frac{EI(4+\phi)}{L(1+\phi)} & 0 & -\frac{6EI}{L^2(1+\phi)} & \frac{EI(2-\phi)}{L(1+\phi)} \\ -\frac{AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & -\frac{12EI}{L^3(1+\phi)} & -\frac{6EI}{L^2(1+\phi)} & 0 & \frac{12EI}{L^3(1+\phi)} & -\frac{6EI}{L^2(1+\phi)} \\ 0 & \frac{6EI}{L^2(1+\phi)} & \frac{EI(2-\phi)}{L(1+\phi)} & 0 & -\frac{6EI}{L^2(1+\phi)} & \frac{EI(4+\phi)}{L(1+\phi)} \end{bmatrix} \quad (14.3\text{–}1)$$

where:
- A = cross–section area (input as AREA on **R** command)
- E = Young's modulus (input as EX on **MP** command)
- L = element length
- I = moment of inertia (input as IZZ on **R** command)
- ϕ = $\frac{12EI}{GA^sL^2}$
- G = shear modulus (input as GXY on **MP** command)
- A^s = $\frac{A}{F^s}$ = shear area
- F^s = shear deflection constant (input as SHEARZ on **R** command)

The element mass matrix in element coordinates for **LUMPM**,OFF is (Yokoyama(167)):

$$[M_\ell] = (\rho A + m)\, L \left(1 - \epsilon^{in}\right) \begin{bmatrix} 1/3 & 0 & 0 & 1/6 & 0 & 0 \\ 0 & A(r,\phi) & C(r,\phi) & 0 & B(r,\phi) & -D(r,\phi) \\ 0 & C(r,\phi) & E(r,\phi) & 0 & D(r,\phi) & -F(r,\phi) \\ 1/6 & 0 & 0 & 1/3 & 0 & 0 \\ 0 & B(r,\phi) & D(r,\phi) & 0 & A(r,\phi) & -C(r,\phi) \\ 0 & -D(r,\phi) & -F(r,\phi) & 0 & -C(r,\phi) & E(r,\phi) \end{bmatrix} \quad (14.3\text{–}2)$$

where: ϱ = density (input as DENS on **MP** command)

m = added mass per unit length (input as ADDMAS on **R** command)

ε^{in} = prestrain (input as ISTRN on **R** command)

$$A(r,\phi) = \frac{\frac{13}{35} + \frac{7}{10}\phi + \frac{1}{3}\phi^2 + \frac{6}{5}(r/L)^2}{(1+\phi)^2}$$

$$B(r,\phi) = \frac{\frac{9}{70} + \frac{3}{10}\phi + \frac{1}{6}\phi^2 - \frac{6}{5}(r/L)^2}{(1+\phi)^2}$$

$$C(r,\phi) = \frac{\left(\frac{11}{210} + \frac{11}{120}\phi + \frac{1}{24}\phi^2 + \left(\frac{1}{10} - \frac{1}{2}\phi\right)(r/L)^2\right)L}{(1+\phi)^2}$$

$$D(r,\phi) = \frac{\left(\frac{13}{420} + \frac{3}{40}\phi + \frac{1}{24}\phi^2 - \left(\frac{1}{10} - \frac{1}{2}\phi\right)(r/L)^2\right)L}{(1+\phi)^2}$$

$$E(r,\phi) = \frac{\left(\frac{1}{105} + \frac{1}{60}\phi + \frac{1}{120}\phi^2 + \left(\frac{2}{15} + \frac{1}{6}\phi + \frac{1}{3}\phi^2\right)(r/L)^2\right)L^2}{(1+\phi)^2}$$

$$F(r,\phi) = \frac{\left(\frac{1}{140} + \frac{1}{60}\phi + \frac{1}{120}\phi^2 + \left(\frac{1}{30} + \frac{1}{6}\phi + \frac{1}{6}\phi^2\right)(r/L)^2\right)L^2}{(1+\phi)^2}$$

r = $\sqrt{\frac{I}{A}}$ = radius of gyration

The element mass matrix in element coordinates for **LUMPM**,ON is:

$$[M_\ell] = \frac{(\rho A + m)\,L\,(1-\epsilon^{in})}{2} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{14.3–3}$$

The element pressure load vector in element coordinates is:

$$\{F_\ell^{pr}\} = \lfloor P_1 \quad P_2 \quad P_3 \quad P_4 \quad P_5 \quad P_6 \rfloor^T \tag{14.3–4}$$

For uniform lateral pressure,

$$P_1 = P_4 = 0 \tag{14.3–5}$$

$$P_2 = P_5 = -\frac{PL}{2} \tag{14.3–6}$$

$$P_3 = -P_6 = -\frac{PL^2}{12} \tag{14.3–7}$$

where: P = uniform applied pressure (units = force/length) (input on **SFE** command)

Other standard formulas (Roark(48)) for P_1 through P_6 are used for linearly varying loads, partially loaded elements, and point loads.

14.3.2 Stress Calculations

The centroidal stress at end i is:

$$\sigma_i^{dir} = \frac{F_{x,i}}{A} \tag{14.3–8}$$

where: σ_i^{dir} = centroidal stress (output quantity SDIR)

$F_{x,i}$ = axial force (output quantity FORCE)

The bending stress is

$$\sigma_i^{bnd} = \frac{M_i\,t}{2I} \tag{14.3–9}$$

where: σ_i^{bnd} = bending stress at end i (output quantity SBEND)

M_i = moment at end i

t = thickness of beam in element y direction (input as HEIGHT on **R** command)

The presumption has been made that the cross–section is symmetric.